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Public money as a store of value, heterogeneous beliefs and banks: implications of CBDC

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Abstract

The bulk of cash is held for store of value purposes, with such holdings sharply increasing in times of high economic uncertainty and only a fraction of the population choosing to hoard cash. We develop a Diamond and Dybvig model with public money as a store of value and heterogeneous beliefs about bank stability that accounts for this evidence. Only consumers who are sufficiently pessimistic about bank stability hold cash. The introduction of a central bank digital currency (CBDC) as a store of value lowers the storage cost of public money and induces partial bank disintermediation, which is nevertheless mitigated by an increase in relative maturity transformation. This has heterogeneous welfare consequences across the population. While cash holders always benefit by switching to CBDC, each of all other consumers may be better off or not depending on the probability of a bank run, her (and all others') belief about such probability and the degree of technological superiority of CBDC.

Keywords: Cash hoarding, central bank digital currency, disagreement, uncertainty shocks, flight-to-safety, bank stability, welfare.
JEL Codes: E41, E58, G11, G21

1 Introduction

In recent years, the use of digital payment methods for transactions has been increasing at the expense of cash, a pattern that has become more pronounced since the outbreak of the Covid-19 crisis (see, e.g., Auer et al. 2020). In response to this shift, central banks have started to investigate the benefits and implications of issuing digital public money or retail central bank digital currency (henceforth CBDC).[1] The ultimate goal of CBDCs is to ensure that individuals operating in an increasingly digitalized economy continue having access to public money as a means of payment. However, due to the perceived substitutability between bank deposits and CBDC (Burlon et al., 2023), there are also concerns that CBDCs may disintermediate banks by being widely used as a store of value (see, e.g., ECB 2020a; FED 2022; BoE 2023). Based on the grounds of existing evidence on cash hoarding, there are good reasons for these concerns. In a recent literature review on the use and holdings of cash, Shy (2023) documents that: (i) the bulk of cash is held for store of value purposes, (ii) cash holdings sharply increase in times of high economic uncertainty and perceived bank instability, and (iii) a significant proportion of the population holds cash as a store of value.

Our main contribution is twofold. First, we develop a model a la Diamond and Dybvig (1983) that accounts for these stylized facts on cash as a store of value by relying on the empirical literature that studies how portfolio choices depend on heterogeneous individual beliefs about the state of the economy (see, e.g., Giglio et al. 2021; Meeuwis et al. 2022). Second, we use this model to study how adopting a CBDC as a storage technology affects bank intermediation - structurally and cyclically (in response to uncertainty shocks) - and welfare by altering consumers' portfolio choice between public (outside) and private (inside) monies. So far, the bulk of the CBDC literature has focused on studying the benefits and costs of CBDC in its role as a means of payment, rather than as a store of value.[2]

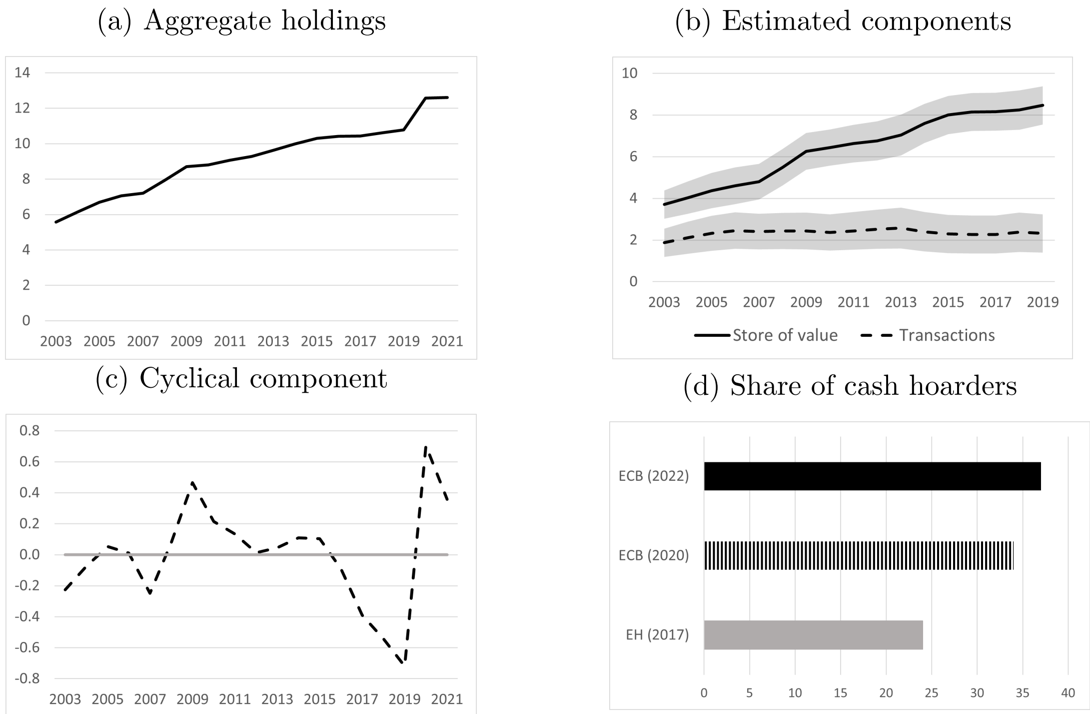
The above outlined empirical regularities on cash are visible in economies all over the world. However, our study is inspired by the case of the euro area, in which these patterns are particularly pronounced and where a key part of the policy debate on how to design CBDCs to limit its use as financial investment instruments has unfolded (Bindseil 2020; Bindseil and Panetta 2020; Adalid et al. 2022). Figure 1 illustrates these empirical observations for the case of the euro area. Panel (a) reports euro-denominated aggregate cash holdings as a percent of GDP at annual frequency for the period 2003-2021. Cash holdings have been increasing over the entire horizon. Panel (b) displays the estimated component of total cash holdings used for transaction purposes (dashed line) and that

[1]See Kosse and Mattei (2022) for the findings of the latest BIS survey on central banks' views and plans regarding CBDCs, and Panetta (2022) for the key differences between retail and wholesale central bank digital currency.

[2]For recent reviews of the literature on CBDC, see Ahnert et al. (2022) and Infante et al. (2022).

used as store of value (solid line).[3] The steady increase in aggregate cash holdings over time is driven by estimated cash holdings as a store of value.[4] Panel (c) plots the cyclical component of euro-denominated aggregate cash holdings, which prominently went up around the Great Recession (2009) and the COVID-19 crisis (2020-2021). Panel (d) reports the share of survey respondents who kept extra cash at hand for store of value purposes in 2016, 2019 and 2021. It suggests that it is not only aggregate cash holdings but also the share of respondents hoarding cash that has been increasing over time and, arguably, jumping in times of high economic uncertainty.

Figure 1: Euro denominated cash holdings



Notes: Panels (a), (b) and (c) refer to cash holdings defined as the value of euro-denominated banknotes in net circulation as a percent of GDP, at annual frequency. Panel (d) reports the share of survey respondents who held euro-denominated cash for store of value purposes in the euro area in 2016, 2019 and 2021 according to survey results published by the ECB in 2017, 2020 and 2022, respectively. Variables represented in panels (a), (b) and (d) are expressed in percentage points. The one plotted in panel (c) is expressed in percentage deviations from the HP trend with a standard smoothing parameter of 100. Data and sources: ECB statistics, Esselink and Hernández (2017), ECB (2020b), ECB (2022), Zamora-Pérez (2021) and own calculations.

In Section 2, we present a version of the Diamond and Dybvig model augmented with cash, which we refer to as the baseline model. As a storage technology, this safe liquid asset serves as an alternative to bank deposits. Banks provide insurance for idiosyncratic liquidity shocks, which exposes the bank to the possibility of a bank run. As in Cooper

[3] Appendix C describes how we decompose aggregate cash holdings into the two components.

[4] Lippi and Secchi (2009) find that this upward trend in the ratio of cash to GDP is visible around the world, both in high and low income countries. The fact that the share of cash transactions has been declining in recent years while aggregate demand for currency has kept growing is commonly referred to as the paradox of cash (Jiang and Shao, 2020). Interestingly, it is not only households but also firms that have been contributing to this trend (Bates et al. (2009).

and Ross (1998); Ennis and Keister (2006), the probability of the bank run equilibrium is exogenously determined according to an equilibrium selection rule. Such exogenous probability captures bank stability and the state of the economy, and is assumed to be public information. Importantly, we interpret the short-term asset in which banks invest as "reserves" (i.e., digital public money that can only be accessed by banks). To capture the technological superiority of reserves when compared to cash (due to the digital nature of the former), we normalize to zero any storage costs the former might be subject to whereas we assume that cash storage costs are strictly positive.

The baseline model does not account for the above described evidence. We prove that regardless of the probability of a bank run, there is no demand for cash in equilibrium since hoarding cash is strictly dominated by the run-proof deposit contract fully backed by reserves. For the same reason, there is no role for CBDC in this model. These findings follow from our assumption that, both consumers and the central bank face an adverse selection problem which impedes them to invest in the long-term asset (i.e., lending). By making this empirically relevant assumption (i.e., asymmetric information faced by central banks and their related risk management frameworks), our analysis rules out the possibility of having a central bank deposit monopoly (Fernández-Villaverde et al., 2021).

In Section 3, we modify the baseline model to introduce heterogeneous beliefs about the probability of the bank run equilibrium. We refer to this set-up as "The Model". The objective probability of a bank run is unknown and consumers draw their ex ante beliefs from a distribution function. Belief disagreement captures the level of economic uncertainty. Consistent with the literature, the bank offers a single deposit contract that maximizes the expected utility of its depositors, which depends on the average belief of depositors. Subsequently, consumers choose between cash and deposits based on their individual beliefs about bank stability. Those consumers who are sufficiently pessimistic about bank stability place their endowment in cash. We show that, due to this novel modification of the Diamond and Dybvig set-up, The Model can account for the main empirical observations on cash hoarding.

In Section 4, we assume that the central bank issues - along with cash and reserves - an unremunerated (retail) CBDC. Due to its digital nature, CBDC is also technologically superior to cash. However, it differs from reserves in that consumers can directly place their endowment in it. To simplify, we assume that CBDC alters neither the probability of a bank run nor individual beliefs about such probability. CBDC lowers the storage cost of holding public money. Hence, all cash holders switch to CBDC and some depositors also opt for switching to CBDC based on their beliefs. Consequently, bank deposits fall. However, lending decreases less than proportionally since the average depositor becomes more optimistic about bank stability. That is, CBDC leads to a "less flighty" depositor base, to which the bank optimally responds by re-balancing its portfolio towards a larger share of lending. Thus, the introduction of CBDC as a store of value in The Model yields a non-trivial trade-off between a favourable decline in the storage costs of public money and an undesirable reduction in lending, which is nevertheless mitigated by an increase in relative maturity transformation. Additionally, we show that in our framework CBDC may amplify or mitigate flight-to-safety in response to uncertainty shocks depending on its technological superiority and the distribution of individual beliefs.

In Section 5, we adopt a normative perspective to illustrate how this trade-off can translate into welfare outcomes under a wide-range of policy-relevant scenarios. The attractiveness of hoarding public money increases with the adoption of CBDC as a store of value, which has heterogeneous welfare consequences across the population. Cash holders always benefit by fully switching to CBDC. Each of all other consumers (i.e., depositors who switch to CBDC and those who remain being depositors) benefit or not depending on the measure of bank stability on which individual welfare depends (i.e., the true probability of a bank run or individual beliefs) and its level, the distribution of individual beliefs about bank stability, and the degree of technological superiority of CBDC. Depending on these factors and on the welfare criterion, CBDC as a store of value may increase social welfare or not, both structurally and in response to uncertainty shocks.

Related literature

The first part of this paper studies how public money that serves as an alternative storage technology to bank deposits can alter (or not) the equilibrium of the standard Diamond and Dybvig model. In a similar model. Allen et al. (2014) also introduces fiat money but that solely serves as a means of payment to study the implications of signing nominal (rather than real) deposit contracts for the equilibrium. More similar to ours, Peck and Setayesh (2022) considers an alternative to bank deposits but in their set-up such option is modeled as a productive investment technology rather than as a storage technology. Our closest antecedent is, perhaps, Ennis and Keister (2003): In order to self-insure themselves against receiving nothing in the event of a bank run, in their model consumers have incentives to place part of their wealth in a safe liquid asset. In our model, this motive for cash hoarding is absent since all depositors who withdraw receive an equal payout in the event of a bank run. Instead, consumers place part of their wealth in a safe liquid asset (i.e., public money) due to the heterogeneity of individual beliefs about bank stability and the state of the economy. To the best of our knowledge, our extension of the standard Diamond and Dybvig model is the first to account for the main stylized facts on cash as a store of value and it does so by relying on the empirical evidence on heterogeneous beliefs and portfolio choice.

This paper connects to the literature on heterogeneous beliefs and disagreement. Heterogeneous beliefs are commonly used in behavioral finance and asset pricing to explain portfolio choices and reactions in financial markets (Hong and Stein, 2007; Chang et al., 2021). Giglio et al. (2021) use survey data to provide robust evidence on: (i) the link between beliefs and portfolio allocations, both across retail investors and over time, and (ii) a persistent heterogeneity in beliefs across individuals. It is well documented that different views on interpreting signals lead to persistent disagreement over economic variables (Harris and Raviv, 1993; Kandel and Pearson, 1995; Meeuwis et al., 2022). Patton and Timmermann (2010) shows that even professional forecasters persistently disagree with a belief dispersion that is counter-cyclical and highest in times of economic recession and uncertainty. Papers in the macro-finance literature that feature heterogeneous beliefs include Geanakoplos (2010); Scheinkman and Xiong (2003); Martin and Papadimitriou (2022); Caballero and Simsek (2020); Shen and Zou (2023). In our model, portfolio choices between risky bank deposits and the safe liquid asset (i.e., public money) depend on individual beliefs about bank stability and the state of the economy, such beliefs are heterogeneous across economic agents, and belief disagreement is used as a proxy for aggregate uncertainty. The latter is an assumption commonly made in this literature on the basis of the evidence that economic uncertainty significantly affects disagreement (Bloom, 2014; Bachmann et al., 2013; Bloom et al., 2018)

Our study also relates to the strand of the literature on flight-to-safety (see, e.g., Adrian et al. 2019 and Baele et al. 2020), which we define as investors' shifts from risky to safe assets in response to higher economic uncertainty. In particular, it connects with the part of that literature that explores how uncertainty shocks trigger flight-to-safety episodes (Bhattarai et al. 2020), lead to short episodes of a sharp decline in productive investment and economic activity (Bloom 2009; Basu and Bundick 2017), and induce distributional effects as a consequence of such safe liquid asset hoarding behaviour (Bayer et al. 2019).

For the particular case of cash as a safe liquid asset, Jobst and Stix (2017) documents an empirical relationship between cash holdings and economic uncertainty for a sample of 70 economies over the period 2001-2014. Shy (2023) also documents an increase in cash hoarding during the COVID-19 pandemic crisis motivated by higher economic uncertainty. Using survey data, Stix (2013) finds that the reason why households in European countries hold sizeable shares of their assets in cash at home rather than at banks relates with their lack of trust in banks and their memories of past banking crises. Ashworth and Goodhart (2020) shows that demand for cash particularly increases during financial crises. Baubeau et al. (2021) finds that the fall in bank credit that took place in France during the interwar period was mostly driven by a flight-to-safety by deposits, from private to public money.

The second part of the paper on the consequences of CBDC as a store of value contributes to the growing literature on the implications of CBDC for bank intermediation and the real economy. Brunnermeier and Niepelt (2019); Fernández-Villaverde et al.

(2021) show that, under certain conditions, the consumer's choice between public and private money does not have any allocative or macroeconomic consequences. In practice, such an "equivalence" result may not hold due to the presence of market imperfections and/or regulatory constraints. Such frictions include, among others, bank market power in the deposit market (Andolfatto, 2021; Chiu et al., 2023), central bank collateral requirements (Assenmacher et al., 2021; Burlon et al., 2023), liquidity regulation (Meller and Soons, 2023), and external financing frictions (Whited et al., 2022).

Our model suggests that CBDC as a store of value can increase social welfare by lowering the storage cost of public money and despite a bank disintermediation effect, which is nonetheless mitigated by an increase in relative maturity transformation. Others have suggested that CBDC can increase welfare by improving transaction efficiency (Agur et al., 2022; Keister and Sanches, 2022), decreasing excessive bank market power (Chiu et al., 2023), stabilizing bank lending supply to the real economy (Burlon et al., 2023), modifying the architecture of the monetary system (Niepelt, 2023), or by avoiding incentives problems in private banking (Williamson, 2022).

In particular, our paper is closely related to the part of this strand of the literature that studies the implications of CBDC for banks and the economy through the lens of the Diamond and Dybvig framework (e.g., Schilling et al. 2020; Fernández-Villaverde et al. 2021; Keister and Monnet 2022; Ahnert et al. 2023; Tercero-Lucas 2023). In Fernández-Villaverde et al. (2021), the equivalence result holds and CBDC can lead to a central bank deposit monopoly as the central bank can indirectly engage in long-term lending by signing contracts with investment banks. In our model it is incomplete information that undermines the equivalence result, through two relevant channels: (i) the central bank (and consumers) face an adverse selection problem, which precludes them from investing in long-term loans, and (ii) consumers do not know the objective probability of a bank run. Belief disagreement on whether to hold deposits or cash allows for the model to capture the main stylized facts on public money as a store of value. Different from all other papers in this strand of the literature, ours studies the implications of CBDC for banks and welfare in a version of the Diamond and Dybvig model with cash and heterogeneous beliefs about bank stability that accounts for key empirical findings documented in the literature on cash hoarding, on one hand, and heterogeneous beliefs and portfolio choices, on the other hand.

The rest of the paper is structured as follows. Section 2 presents the baseline model. Section 3 develops The Model by introducing heterogeneous beliefs about the probability of a bank run. Section 4 extends The Model by allowing for CBDC as a store of value. Section 5 performs a welfare analysis. Section 6 concludes.

2 The baseline model

The baseline model extends the Diamond and Dybvig-type banking model of Cooper and Ross (1998); Ennis and Keister (2006) to allow for cash as a store of value.

2.1 Environment

There are three dates $t = 0, 1, 2$ and a single good per date which works as a numeraire and can be used for investment at $t = 0$ and consumption at $t = 1$ and $t = 2$. A unit continuum of ex ante identical consumers indexed by $i \in [0, 1]$ has an endowment normalized to one at $t = 0$. Consumer preferences are given by

$$U(c_1, c_2, \theta_i) = u(c_1 + \theta_i c_2),$$

where c_t is consumption at date t and the utility function u is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada conditions. The idiosyncratic liquidity shock $\theta_i \in \{0, 1\}$ is realized at $t = 1$ and privately observed by each consumer. If $\theta_i = 0$, consumer i is impatient and wishes to consume at the interim date only; otherwise, she is patient and values consumption at either the interim or final date. The probability of each consumer becoming impatient is a constant λ.

Consumers can invest in two types of assets at $t = 0$ to transfer wealth to future dates: retail central bank money ("cash") and bank deposits. Without loss of generality and for the sake of simplicity, we do not allow for mixed portfolios. That is, consumers have to place their entire endowment either in deposits or in cash. Appendix B offers a numerical solution to the consumer's problem in a version of The Model in which a consumer can have a mixed portfolio by simultaneously allocating a positive proportion of her endowment in both, deposits and cash.

There is a central bank that exchanges endowment for cash at $t = 0$ and $t = 1$ and repays consumption goods on demand at $t = 1$ and $t = 2$. While the central bank faces no direct storage costs, holding cash comes with a proportional cost $f > 0$ incurred whenever the cash is exchanged for consumption or any other asset, so a unit of cash has a net exchange value of $1 - f$ whenever used.[5]

Second, consumers can pool resources to form a bank that invests their endowments on their behalf. At $t = 0$ the bank invests an amount x of its deposit funding D_0 received from consumers in a long-term investment technology, and $D_0 - x$ in wholesale central bank money ("reserves"). Reserves can only be accessed by the bank and the net exchange value per unit of reserves is normalized to one.

The long-term investment technology (long-term lending) can be of two types, good

[5]This cost could correspond to resources spent to prevent theft before its conversion or on other storage and transportation costs.

and bad. The good type yields a return of R units upon maturity at $t = 2$ and has no liquidation value at $t = 1$.[6] This technology offers a higher long-term return than cash or reserves, but it is less liquid. The bad type —a lemon—never generates any return, similar to Dang et al. (2017).

Only a bank can screen potential borrowers and prevent investment in the bad technology, as in Holmstrom and Tirole (1997). As in Allen and Gale (1998), we assume that the implied adverse selection problem precludes the consumers and the central bank from investing directly or indirectly (via lending to the bank) in the long-term technology.[7] Thus, the bank has two functions in this economy: (i) it serves as a conduit for investment in good long-term technologies, while screening bad ones, and (ii) it provides insurance against idiosyncratic liquidity risk by offering demand deposits to consumers, as in Diamond and Dybvig (1983). Specifically, at $t = 0$ the bank offers a contract that promises a payment of c_1^B if a consumer withdraws at $t = 1$ and c_2^B if she does not. However, such promises are only fulfilled if the consumers withdrawing at $t = 1$ are the proportion λ of impatient ones. As in Allen and Gale (1998), we assume that if the proportion of early withdrawers exceeds λ, the bank "defaults" and makes a liquidation payment c_R^B to all consumers attempting to withdraw at $t = 1$ (and zero to the rest).[8]

The timing of events in the baseline model is as follows. First, each consumer chooses between holding cash or depositing with the bank. The bank, on behalf of its depositors, invests x in the long-term technology and $D_0 - x$ in reserves. At date $t = 1$, the liquidity shock hits and all impatient consumers attempt to withdraw their bank deposits. The actions of patient consumers depend on: (i) what she expects other patient consumers will do, and (ii) the deposit contract. To simplify the discussion, we will focus on the case in which consumers play symmetric pure strategies. If a patient consumer believes other patient consumers will not withdraw and the deposit contract is incentive compatible, ($c_2^B \geq c_1^B$), she will optimally decide not to withdraw her bank deposits. If all patient consumers follow this behavior, a "good" non-run equilibrium can be sustained. However, if she expects all other patient consumers to withdraw and the bank does not have enough resources to pay c_1^B to all depositors, she will optimally decide to withdraw. If all patient consumers follow this behavior, a "bad" bank run equilibrium emerges. When the bank cannot cover the required repayment in case all patient depositors withdraw at $t = 1$, the deposit contract is said to be run-prone.[9] In contrast, if the bank has enough reserves to

[6] Jacklin and Bhattacharya (1988); Haubrich and King (1990) also assume that the long-term asset yields a zero payoff when liquidated early. In contrast, Diamond and Dybvig (1983) assume that the liquidation value is equal to the initial investment, while Ennis and Keister (2006); Cooper and Ross (1998) include a liquidation cost $\tau \in [0, 1]$.

[7] Arguably, in practice, adverse selection explains, among others, why central bank lending and asset purchases are subject to strict risk management frameworks.

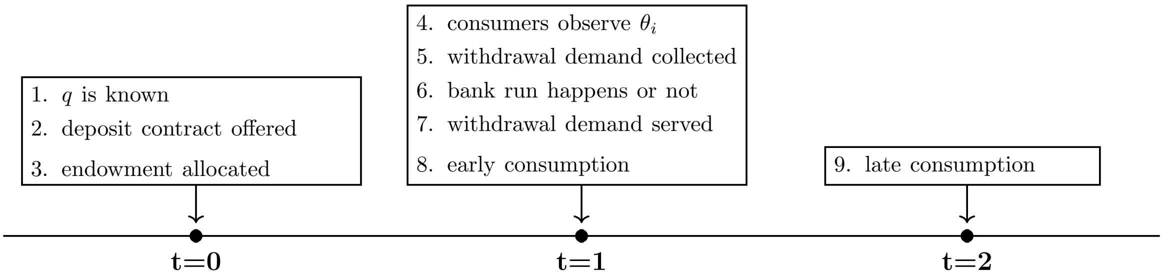
[8] Consumers cannot trade at dates $t = 1$ and $t = 2$. Jacklin (1987) and Wallace (1988) consider a credit market at date $t = 1$.

[9] Given the assumption that the long-term investment technology has no liquidation value at $t = 1$, these resources amount to the reserves held by the bank.

meet all of its short-term obligations, waiting to withdraw is a dominant strategy as the payment at $t = 2$ is larger than the payment at $t = 1$. In that case, the deposit contract is said to be run-proof.

In order to describe the ex ante optimal deposit contract anticipating the possibility of multiple equilibria, we follow Cooper and Ross (1998) and Ennis and Keister (2006) and assume a sunspots-based equilibrium selection rule: if both equilibria exist, a bank run occurs with an exogenous probability q. The probability q is constant and does not depend on actual bank reserves, and we have that $(1 - q)R > 1$. Figure 2 summarizes the timeline of the game.

Figure 2: Timeline of the baseline model



2.2 Optimal demand for cash

To determine the demand for cash, we specify the problem of a bank that behaves competitively in the sense that it offers the contract that maximizes the expected utility of its depositors. Let $\bar{\lambda}$ denote the fraction of depositors that can be served at the interim date under the underlying contract. Variable y represents reserves that are needed to repay impatient depositors whereas y^l represents excess liquidity, i.e. reserves in excess of what is required to repay impatient depositors only. The bank's problem solves

$$\max_{c_1^B, c_2^B, c_R^B, x, y, y^l} (1 - q\mathbb{1}_{\bar{\lambda}<1}) \Big[\lambda u(c_1^B) + (1-\lambda)u(c_2^B)\Big] + q\mathbb{1}_{\bar{\lambda}<1} u(c_R^B) \tag{A}$$

subject to

$$x + y + y^l = D_0, \quad (1) \qquad \lambda c_1^B = y, \quad (2)$$

$$(1-\lambda)c_2^B = Rx + y^l, \quad (3) \qquad c_R^B = y + y^l, \quad (4)$$

$$0 \leq c_1^B \leq c_2^B, \quad (5) \qquad c_1^B, c_2^B, x, y, y^l \geq 0, \quad (6)$$

The indicator function $\mathbb{1}_{\bar{\lambda}<1}$ reflects the equilibrium selection rule. A bank run only occurs with probability q if $\bar{\lambda} < 1$ and otherwise occurs with probability zero. The maximum fraction of depositors that can be served at the interim date without a default is given by

$$\bar{\lambda} = \frac{y + y^l}{c_1^B}. \tag{7}$$

Problem A states that the bank maximizes the expected utility of its depositors subject to the following constraints. Expression 1 stipulates that the bank invest all its deposit funding. The bank is a deposit taker. According to Expression 2, the bank must hold enough reserves to cover the promised interim return. Since there is no aggregate uncertainty, the bank knows that a fraction λ of depositors will have liquidity needs. Expression 3 states that the final payment equals the sum of the return on long-term lending and the remaining reserves after having serviced early withdrawals. Expression 4 dictates that the payment in case of a bank run is equal to the liquidation value of the bank. Expression 5 is the incentive compatibility constraint, which ensures that patient consumers have no incentive to withdraw at the interim date in absence of a bank run.

The optimal deposit contract solves Problem A for payments (c_1^B, c_2^B, c_R^B), the bank asset allocation (x, y), subject to the level of deposit funding, D_0. Therefore, it also implicitly includes the demand for cash ($M_0 = 1 - D_0$).

Proposition 1 shows that regardless of the terms of the optimal deposit contract, there is never a positive demand for cash in the baseline model.

Proposition 1: *In the baseline model, $M^0 = 0$, $\forall\ q \in (0, 1)$.*

The reasoning for Proposition 1 is as follows. A bank, whose objective is to maximize depositor utility, can always offer a run-proof contract for any realization of q. The expected utility obtained from the best run-proof is certainly as high as the expected utility obtained from the run-proof contract that includes only reserve holdings. In turn, the utility obtained from cash holdings is strictly lower than that from a run-proof deposit contract with only reserve holdings due to the storage costs (and the lack of liquidity insurance). Thus, the bank never chooses to offer a deposit contract that is inferior to cash holdings and a consumer never prefers to hold cash instead of bank deposits.

To further characterize the solution to the baseline model, we assume a utility function of the constant-relative-risk-aversion form

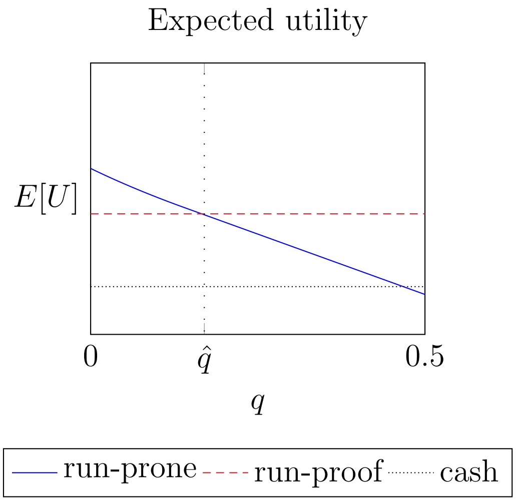
$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{with } \gamma > 1, \tag{8}$$

Corollary 1 defines a cut-off value $\hat{q}$ that determines whether the solution to Problem (A) is a run-prone or a run-proof contract, similar to Proposition 5 in Cooper and Ross (1998).

Corollary 1: *There exists a $\hat{q} \in (0, 1)$ such that if $q > \hat{q}$ the optimal deposit contract is run-proof whereas if $q < \hat{q}$ it is run-prone. Regardless of the probability of a bank run,* *there is no demand for cash in the baseline model.*

The intuition behind the proof contained in Appendix A is the following. When $q = 0$, clearly the optimal contract is the run-prone contract that maximizes expected return by lending long-term. The bank optimally responds to a higher q by substituting long-term loans for additional reserves to increase its liquidation value and thus the payment in case of a bank run. This substitution lowers the expected utility obtained from the run-prone contract. As an alternative, the bank can offer the best run-proof contract, in which case the expected utility is independent from q. When $q > \hat{q}$, the expected utility from the best run-proof contract exceeds that from the best run-prone contract, while when $q < \hat{q}$ a run-prone contract results in higher expected utility. Figure (3) illustrates this result by means of a simulation.

Figure 3: Optimal deposit contract vs cash



The figure plots the expected utility when $R = 2.0$, $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

To summarize, the baseline model fails to explain any of the empirical facts on cash holdings as a store of value outlined in Section 1. Notably, in the baseline model there is no demand for cash regardless of the state of the economy. The next section extends the baseline model to account for these empirical regularities by relying on the literature on portfolio choice and disagreement.

3 The Model

This section extends the baseline model to allow for individual heterogeneous beliefs about the probability of a bank run.

3.1 Heterogeneous beliefs

The baseline model assumes that if multiple equilibria exist, a bank run occurs with an exogenous probability q that is known ex ante by all consumers at $t = 0$ and before they decide on how to allocate their endowment. Consider instead that consumers do not have such information but have heterogeneous beliefs (at $t = 0$) about the probability of a bank run. These beliefs are exogenous.

Formally, a consumer i has belief q_i at $t = 0$ about the probability of a bank run at $t = 1$, if it exists. At $t = 0$ each consumer draws her belief q_i from a cumulative distribution $F(q, \sigma)$ with support $[0, 1]$ and density $f(q, \sigma)$. We assume that a greater σ correlates with greater aggregate belief dispersion in the sense of a mean preserving spread (see Rothschild and Stiglitz 1978; Stiglitz and Weiss 1981), i.e. for $\sigma_1 > \sigma_2$ it holds that

$$\int_0^1 q_i f(q, \sigma_1) dq = \int_0^1 q_i f(q, \sigma_2) dq,$$

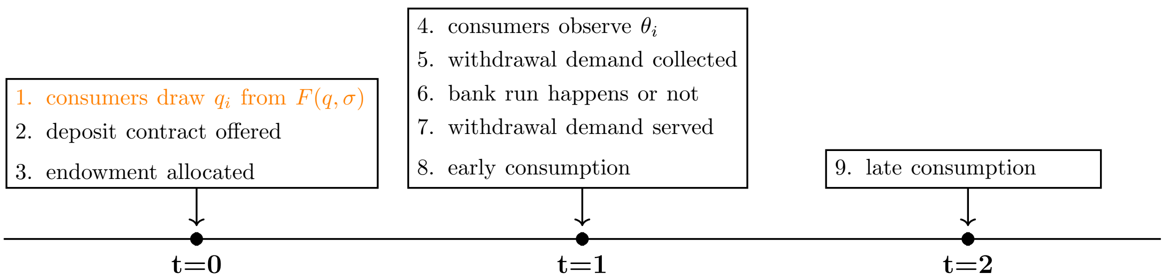
while for any $t > 0$ it holds that

$$\int_0^t F(q, \sigma_1) dq \geq \int_0^t F(q, \sigma_2) dq.$$

Except for their beliefs, consumers remain ex ante identical.

Figure 4 presents the timeline. Importantly, in this set-up consumers make their portfolio choice at $t = 0$ based on their belief q_i about the probability of a bank run at $t = 1$. Note that the baseline model is equal to the case for which $\sigma = 0$ as all consumers agree on the probability q of a bank run.

Figure 4: Timeline of The Model



3.2 Optimal demand for cash

We turn our attention to the banks' problem and the household's portfolio choice, asking under what conditions the optimal demand for cash as a store of value is positive. If the chosen deposit contract is run-proof, individual beliefs q_i are irrelevant and the results presented in Section 2 apply.

If the chosen deposit contract is run-prone, a bank run may occur. Consistent with the literature and the baseline model, we assume that the bank offers a single deposit contract that maximizes the expected utility of its depositors. In particular, the bank offers a deposit contract based on the average individual belief of its depositors. Section 3.4 discusses the case of a representative bank that offers a menu of contracts which allows consumers to self-select the one that best matches their individual beliefs.

Consumers who are sufficiently pessimistic about bank stability (sufficiently high q_i) believe to be better off with cash than with the run-prone deposit contract. Proposition 2 states that if the deposit contract that solves the bank's problem is run-prone and the depositor is sufficiently pessimistic, or $q_i > \tilde{q}$, consumer i prefers to hold cash rather than bank deposits. The threshold value $\tilde{q}$ that defines the set of consumers who prefer to hold public money is given by

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})}, \tag{9}$$

which is the subjective probability of a bank run for which a consumer is indifferent between placing her endowment in bank deposits and placing it in cash.[10]

Proposition 2: *Given a certain run-prone deposit contract: (i) consumers with $q_i > \tilde{q}$ place their endowment in cash, (ii) a proportion $(1-\tilde{q})$ of consumers holds cash, and (iii) $M_0 = \int_{\tilde{q}}^{1} f(q,\sigma)dq$.*

Provided that the deposit contract offered by the bank is run-prone, consumers who are sufficiently pessimistic about bank stability hold cash. Aggregate demand for cash is given by the sum of individual cash holdings for all consumers with $q_i > \tilde{q}$.

Despite the fact that the bank cannot observe individual beliefs about the probability of a bank run, the bound $\tilde{q}$ and thus the fraction of consumers who optimally place their endowment in deposits is known to the bank and depends on the chosen deposit contract. Consequently, the bank solves

$$\max_{c_1^B, c_2^B, c_R^B, x, y} \left[1 - \int_0^{\tilde{q}} q_i f(q,\sigma_1)dq\right]\left[\lambda u(c_E) + (1-\lambda)u(c_L)\right] + \left[\int_0^{\tilde{q}} q_i f(q,\sigma_1)dq\right] u(c_R), \tag{B}$$

subject to the same constraints as Problem (A). Importantly, the beliefs q_i are assumed to be unaffected by the chosen deposit contract.

Denote the average belief of depositors as $\int_0^{\tilde{q}} q_i f(q,\sigma_1)dq = \bar{q}$. Problem (B) results in

[10] Without loss of generality and for the sake of tractability, we have assumed that at $t = 0$ each individual places her endowment either in cash or in deposits. See Appendix C for the general version of the consumer's problem which allows each consumer to simultaneously allocate a positive proportion of her endowment in both deposits and cash (i.e. a mixed portfolio). A simulation shows how consumers with an interior belief choose a mixed portfolio.

the following optimality condition

$$(1-\bar{q})\Big[Ru'(c_B^2) - u'(c_1^B)\Big] = \bar{q}u'(c_R^B). \tag{9}$$

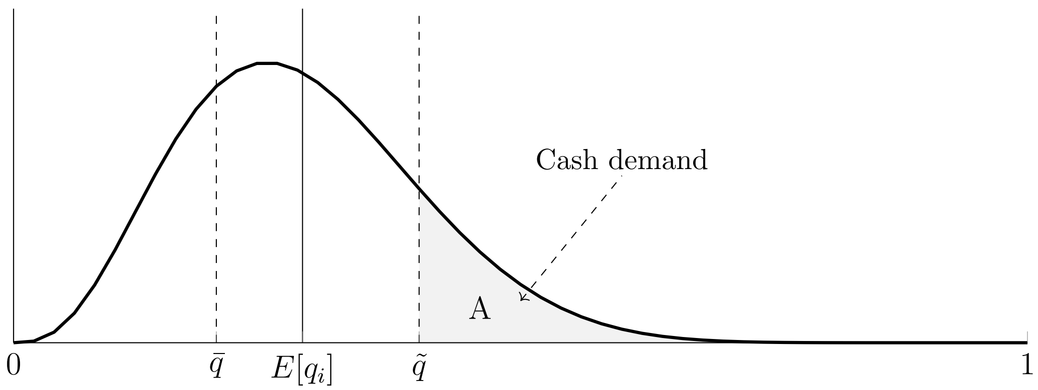
Without loss of generality, Figure 5 uses a Beta distribution with parameters (4,10) to illustrate how, on average, depositors are relatively optimistic about bank stability or $\bar{q} < E[q_i]$.[11] Consumers with $q_i > \tilde{q}$ are cash holders as they believe they are better off holding cash. Aggregate cash holdings are given by shaded region A. Note that $\bar{q}$ is the average belief of those consumers who deposit with the bank, so it must be that $\bar{q} < \tilde{q}$.

Similar to Corollary 1, Corollary 2 defines a cut-off value $\hat{\hat{q}}$ that determines whether the solution to Problem (B) is a run-prone or a run-proof contract under the assumption that Expression (8) applies.[12] The bank offers a run-prone contract when the average beliefs of its depositors $\bar{q}$ is sufficiently low.

Corollary 2: *There exists a $\hat{\hat{q}} \in (0,1)$ such that if $\bar{q} > \hat{\hat{q}}$ the solution to Problem (B) is a run-proof contract and if $\bar{q} < \hat{\hat{q}}$ it is a run-prone contract.*

Corollary 2 follows from the proof of Corollary 1, which applies for all D_0. The difference between $\hat{\hat{q}}$ and $\hat{q}$ is due to the difference between $\bar{q}$ and q in the bank's objective function, which relates to the existence of a demand for cash and, ultimately, to the presence of heterogeneous beliefs about bank stability.

Figure 5: Aggregate demand for cash



Notes: The figure plots the distribution of beliefs when $q_i \sim Beta(4, 10)$.

[11] This result applies to all distributions implying a run-prone contract.

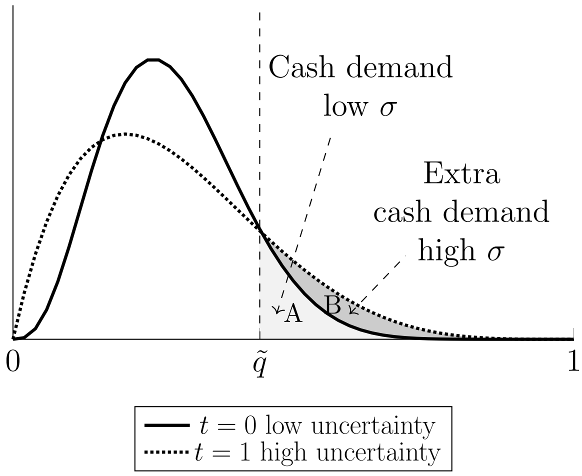
[12] The remainder of the analysis continues assuming that this specification of the utility function applies.

3.3 Uncertainty and demand for cash

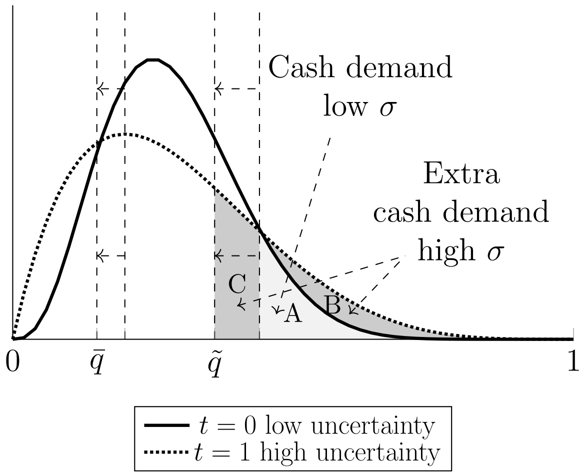
Then, we study how uncertainty affects demand for cash and the deposit contract offered by the bank. We define flight-to-safety as a shift from the risky (i.e., deposits) to the safe asset (i.e., public money) in response to an uncertainty shock. In The Model, aggregate uncertainty is captured by the dispersion in individual beliefs, σ, and uncertainty shocks are defined as exogenous mean-preserving spread shifts in σ at $t = 1$, before consumers observe θ_i.

Provided that the contract offered by the bank is run-prone (i.e., $\bar{q} < \hat{\hat{q}}$) and the majority of consumers are depositors (i.e., $\tilde{q} > E[q_i]$), the economy partially shifts from deposits to cash in response to higher uncertainty. Figure 6 illustrates how The Model captures flight-to-safety episodes and how they can be decomposed into a direct and an indirect effect. First, for any given run-prone deposit contract, an increase in beliefs' dispersion leads to an increase in aggregate demand for cash as the mass of consumers in the tails of the distribution increases (region B).[13] Second, as depositors on average perceive a bank run to be less likely (i.e., $\bar{q}$ declines), the bank adjusts its own liquidity risk profile by increasing the share of long-term lending in its portfolio.[14] The liquidation value of the bank decreases and so does the bound $\tilde{q}$. As a result, consumers in region C also switch to cash. Proposition 3 summarizes the main implications of an increase in beliefs' dispersion, σ, for cash demand and the optimal run-prone deposit contract.

Figure 6: Response in cash demand to uncertainty shocks



(a) Direct effect: Upper tail becomes fatter



(b) Indirect effect: $\bar{q}$ and $\tilde{q}$ shift downwards

Notes: The figures illustrate the demand for cash when at $t = 0$ the distribution of beliefs is $q_i \sim Beta(4, 10)$ (low uncertainty) and at $t = 1$ it is $q_i \sim Beta(2, 5)$ (dotted line, high uncertainty).

Proposition 3: *Given a certain run-prone contract ($\bar{q} < \hat{\hat{q}}$) and a population with a*

[13] Recall from Section 3.1 that a greater value of σ implies a greater dispersion in beliefs but does not affect the mean of the distribution.

[14] Optimality condition (9) indicates that the share of bank reserves, y, is strictly decreasing in $\bar{q}$.

majority of depositors ($\tilde{q} > E[q_i]$), as σ increases: (i) cash demand increases, (ii) bank deposits and the average belief of depositors $\bar{q}$ decrease, (iii) the bank reduces reserve holdings and long-term lending, (iv) the bank reduces the share of reserves in its portfolio $\frac{y}{D_0}$, and (v) the bound $\tilde{q}$ decreases.

The interpretation of Proposition 3 is as follows. The number of consumers who prefer to hold cash (rather than bank deposits) increases with aggregate uncertainty. As a result, the remaining depositors are - on average - more confident about bank stability. The bank optimally responds to this shift in the average belief of depositors by offering a relatively higher payment in the good equilibrium and a relatively lower payment in case of a bank run. It does so by re-balancing its asset portfolio towards more long-term lending, which increases relative maturity transformation.

In a nutshell, The Model accounts for key empirical findings on cash holdings as a safe liquid asset: (i) at the aggregate level there is a demand for cash as a store of value, (ii) only a certain proportion of consumers hold cash (i.e., those who are sufficiently pessimistic about bank stability and the future state of the economy), and (iii) aggregate demand for cash and the proportion of consumers who hold public money for safety reasons increase with economic uncertainty.

3.4 Discussion

Our study builds on several key assumptions, which are empirically relevant and deserve some discussion. First, our assessment on the effects of uncertainty shocks assumes the majority of consumers are depositors ($\tilde{q} > E[q_i]$). If instead the majority of the population were cash holders, a mean-preserving shock to belief dispersion would translate into an increase in the share of optimistic consumers (i.e., having a belief below $\tilde{q}$). That would, ultimately, lead to an increase (rather than a decline) in bank deposits.

This raises the question why we model an uncertainty shock as a mean-preserving shock to σ. The advantage of this approach is that it allows to interpret flight-to-safety as being solely caused by higher disagreement. An alternative would be to model uncertainty shocks as shifts in the entire distribution of beliefs (mean and dispersion). While this model choice would also allow to capture shifts from deposits to cash, it would no longer be clear whether they can be regarded as pure flight-to-safety episodes, as such shifts could be driven not only by uncertainty (captured by belief dispersion) but also by shifts in the average belief of consumers.

Second, our assessment on the effects of uncertainty shocks only considers equilibria in which the bank offers a run-prone contract ($\bar{q} < \hat{\hat{q}}$) and ignore the cases in which, due to a high average belief of depositors, the optimal contract offered by the bank is run-proof. In those equilibria, there would be no role for the bank run equilibrium and there would be no disagreement. All consumers would place their endowment in the run-proof contract and there would be no cash holdings. An uncertainty shock would have no implications.

Third, the bank offers only one contract which is based on the average belief of depositors. Alternatively, consider a representative bank that offers a menu of deposit contracts or, alternatively, a continuum of banks each of them offering a different deposit contract based on specific individual beliefs. In equilibrium this would result in a continuum of run-prone deposit contracts adapted to differing individual beliefs. Each optimistic consumer self-selects the optimal run-prone deposit contract that matches her individual belief (with $\bar{q} = q_i$).

In case of a continuum of deposit contracts being offered - that simultaneously include run-prone and run-proof deposit contracts - not only would all optimistic consumers self-select their optimal run-prone contract, but also the pessimistic consumers - for whom $q_i < \hat{q}$ (see Corollary 1) - would self-select the unique optimal run-proof deposit contract. Recalling from Proposition 1 that the optimal run-proof deposit contract is preferred over cash, it follows that this extension would yield a separating equilibrium with no demand for public money as a store of value.[15]

Fourth, for simplicity we assume that consumers do not update their beliefs. Consider, instead, that they learn and revise their beliefs based on available information. By observing the deposit contract offered by the bank, consumers can infer the average belief of depositors. In such an environment, they would update their individual beliefs, which in turn would affect the average belief of depositors itself. This type of higher order belief formation would result in less disagreement among consumers. In the limit, individual beliefs would converge, there would be no disagreement and this version of The Model would nest the baseline model. However, as long as consumers only partially update their beliefs based on the average belief of depositors, there would be some disagreement about bank stability and our results would remain qualitatively unaffected. In this regard, it is worth noting that consumers can never learn the objective probability of a bank run since it is unknown to all agents in this environment.

4 The Model with CBDC

This section extends the Model to allow for the central bank to issue central bank digital currency (CBDC) as a store of value along with cash and reserves. The study is restricted to the equilibria in which the optimal deposit contract is run-prone.

[15]In this discussion we refer to an extension in which pessimistic consumers believe that the bank offering the run-proof contract is not subject to any risk of default (e.g., a narrow bank). A full treatment of this extension would ideally incorporate incentives problems in private banking and wider belief formation.

4.1 CBDC vs cash

As for the case of cash, the central bank exchanges endowment for CBDC at $t = 0$ and $t = 1$ and repays consumption goods on demand at $t = 1$. Inspired by the ongoing policy debate, we allow for CBDC to theoretically differ from cash along three dimensions. First, the interest rate on CBDC, r^{DC}, can be equal to zero or negative.[16] Second, the authority can impose a quantity limit on CBDC holdings. Third, CBDC storage costs, f^{DC}, may differ from cash storage costs, f.

For any given run-prone contract offered by the bank, a consumer prefers to hold cash or CBDC depending on the exchange value of each of the two forms of public money. Proposition 4 summarizes this choice.

Proposition 4: *A consumer strictly prefers to hold CBDC rather than cash if* $(1+r^{DC} - f^{DC}) > (1 - f)$.

Under a run-prone deposit contract, Proposition 4 has several implications. First, by adequately calibrating r^{DC}, the central bank can determine whether consumers prefer to hold cash or CBDC as a store of value. Second, by introducing a limit on CBDC holdings $\bar{M}^{DC} < M_0$, where $\bar{M}^{DC}$ denotes the individual CBDC quantity limit, the central bank can calibrate the aggregate amount of CBDC held as a store of value. Third, if the only difference between CBDC and cash is given by $f > f^{DC}$, CBDC fully replaces cash as a safe store of value. In the remainder of the paper we make this assumption. That is, CBDC is a technologically superior public storage technology that is subject neither to negative remuneration nor to quantity limits. As we do throughout the entire paper for the case of reserves, in our simulations the digital nature-driven technological superiority of CBDC is captured by normalizing f^{DC} to zero.

4.2 CBDC vs deposits

The introduction of a CBDC may also affect the run-prone contract offered by the bank and, ultimately, the store of value choice made by consumers. In other words, the issuance of a CBDC in The Model may affect both, the demand for public money as well as bank intermediation.

Under the assumption that CBDC and cash only differ in that $f > f^{DC}$, the threshold for q that defines the set of consumers who prefer to hold public money is no longer given by $\tilde{q}$ as it depends on f^{DC} rather than on f (recall Expression 9). We find that $\tilde{\tilde{q}} < \tilde{q}$ and $\bar{\bar{q}} < \bar{q}$, where $\tilde{\tilde{q}}$ and $\bar{\bar{q}}$ are the threshold with CBDC. Proposition 5 summarizes the main implications of this result.

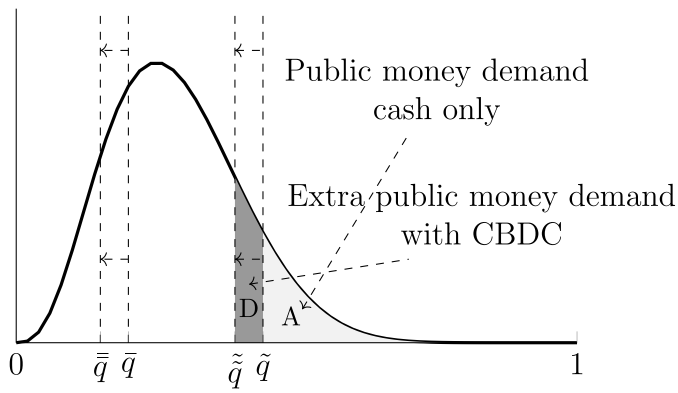
Proposition 5: *Given a CBDC that only differs from cash in* $f > f^{DC}$*, CBDC: (i)*

[16] In The Model with CBDC the interest rate on CBDC holdings cannot be strictly positive since the central bank would not have revenues to cover related expenses.

decreases the threshold that defines the set of consumers who prefer to hold public money ($\tilde{\tilde{q}} < \tilde{q}$), (ii) increases demand for public money, M_0, (iii) reduces bank deposits, D_0 and the average belief of depositors ($\bar{\bar{q}} < \bar{q}$), (iv) decreases reserves and long-term lending, and (v) reduces the share of reserves in the bank's portfolio, $\frac{y}{D_0}$.

Intuitively, the introduction of a superior public storage technology leads to a reduction in the threshold that defines the set of consumers who prefer to hold public money. That is, there is a positive fraction of consumers who switch from bank deposits to CBDC on the basis of their pre-existent beliefs, as represented by region D in Figure 7. This results in a decline in bank deposit funding. The corresponding decrease in long-term lending is less than proportional (i.e., increased relative maturity transformation); remaining depositors are - on average - more optimistic about bank stability and, hence, the representative bank optimally increases the share of long-term lending.

Figure 7: Aggregate demand for CBDC



Notes: The figure plots the distribution of beliefs when $q_i \sim Beta(4, 10)$.

Proposition 5 implies that the deposit contract offered by the bank changes after the introduction of CBDC. As stated in Corollary 3, banks will offer a lower return at the interim date to depositors who withdraw early and a higher return at the final date to patient depositors.

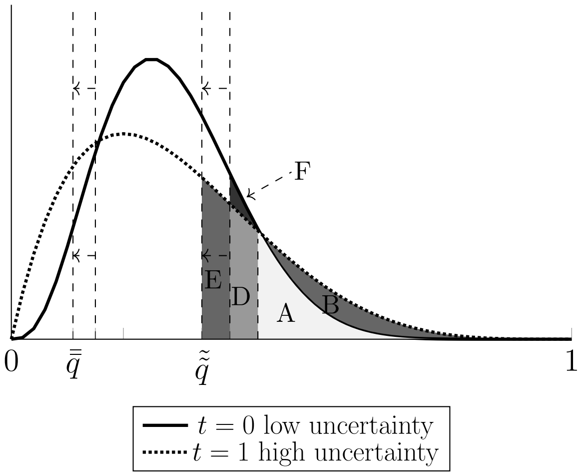
Corollary 3: *CBDC: (i) reduces the return on deposits at the interim date, $\frac{c_1^B}{D_0}$, and (iii) increases the return on deposits at the final date, $\frac{c_2^B}{D_0}$.*

4.3 Uncertainty and demand for CBDC

How does CBDC affect flight-to-safety as defined in Section 3.3? Our study suggests that it can amplify or mitigate it depending on the distribution of individual beliefs and the relative attractiveness of holding CBDC.

Figure 8 assumes the same belief distributions used in Section 3.3 to illustrate the dimensions along which the scale of flight-to-safety in The Model with CBDC can differ from that in The Model. The direct and indirect effects of uncertainty shocks still apply in the former. However, while the direct effect remains unaffected (region B), the indirect effect may differ due to two main reasons (regions E and F of Figure 8 vs region C of Figure 6). First, region D makes clear that, at the time of the shock, the mass of depositors and thus the average belief of depositors and the deposit contract offered by the bank are different in the two models. Second, there is a difference in the storage costs on public money holdings given by (and increasing in) parameter f. In other words, the net return on bank deposits and public money both vary across the two models. CBDC amplifies the scale of flight-to-safety if and only if $E - F > C$. This inequality may hold or not depending on the change in the relative net return on CBDC.

Figure 8: CBDC demand after a shock to uncertainty



Notes: The figure illustrates the demand for CBDC when at $t = 0$ the distribution of beliefs is $q_i \sim Beta(4, 10)$ (low uncertainty) and at $t = 1$ it is $q_i \sim Beta(2, 5)$ (dotted line, high uncertainty) for the case with CBDC.

The intuition of why CBDC as a store of value can potentially amplify or mitigate the scale of flight-to-safety relates to the trade-off introduced by the technological superiority of CBDC, captured by f. On the one hand, a decline in the cost of storing public money potentially increases the scale of flight-to-safety as it becomes more likely that the most pessimistic depositors will be better-off by switching to CBDC in response to the shock. On the other hand, the more prominent the technological superiority of CBDC is, the larger the proportion of depositors who have switched to CBDC before the shock (i.e., when CBDC was introduced) is and hence the smaller the proportion of total consumers who continue being depositors (and can potentially fly to the safe liquid asset in response to the shock). Appendix D presents simulations that illustrate how the sign and scale of the excess flight-to-safety induced by CBDC, given by $E-F-C$, vary across distributions of individual beliefs and the range of f values.

4.4 Discussion

Our CBDC analysis builds on a set of specific assumptions. First, parameter f^{DC} stands for CBDC storage costs. More generally, this parameter could be interpreted as a proxy for all CBDC design features other than remuneration and quantity limits (already captured by r^{DC} and $\bar{M}^{DC}$), such as privacy and accessibility, among many others. This implies that, even if from a pure storage perspective CBDC should be more efficient than cash, depending on the actual design of CBDC it could be the case that $f^{DC} > f$. In that case, according to The Model there would be no role for CBDC as a store of value.

The Model also assumes that the probability of a bank run and the distribution of individual beliefs about such probability are exogenous. This implies that none of them can be affected by CBDC, which may be regarded as unrealistic. An inspection of two possible extensions of The Model suggests that the net endogenous impact of CBDC on any of these variables would be non-trivial. Ahnert et al. (2023) adopt a global games approach to study the impact of CBDC on the probability of a bank run. The mechanisms that would arguably come into play in The Model are similar to theirs. On the one hand, the increase in the net return on public money holdings (which in the case of The Model occurs via a decline in storage costs) increases depositors' incentives to withdraw. On the other hand, it also induces the bank to offer a more attractive contract by increasing the return on deposits at the final date in an effort to retain funding (see Corollary 3).

Something similar can be argued for the case of the net impact of CBDC on the distribution of beliefs, for instance when learning is assumed in a repeated game. On the one hand, if CBDC makes the economy more prone to bank runs, consumers would tend to become more pessimistic over time, as they experience more bank failures. On the other hand, the fact that issuing CBDC means having a more optimistic depositor base on average could move society as a whole to become more optimistic about bank stability.

5 Welfare

This section illustrates the individual and social welfare implications of introducing CBDC in The Model. As in previous sections, our analysis only considers equilibria in which the optimal deposit contract is run-prone, the majority of consumers are depositors ($\tilde{q} > E[q_i]$), the only source of heterogeneity across consumers is their individual beliefs, q_i, and the only difference between CBDC and cash is given by $f > f^{DC}$.

There is no generally accepted approach to welfare analysis in the case of heterogeneous beliefs (e.g. see Brunnermeier et al. 2014; Dávila and Schaab 2022). For instance, does individual welfare depend on actual bank stability, on individual perceptions about bank stability, or on both? Should welfare of all individuals weigh equally in the social welfare function or not? We consider an utilitarian social planner who maximizes a measure of social welfare by choosing between introducing CBDC as a store of value and not doing so. We differentiate between two general cases. In "Case 1" individual welfare of consumer i depends on the true probability of a bank run, q^{true}. In "Case 2" individual welfare of consumer i depends on her individual belief, q_i. In each case, we further distinguish between a sub-case "A" in which the social planner has all relevant information to maximize social welfare and a sub-case "B" in which she does not have all relevant information and the measure of social welfare that she maximizes is her own estimate.

Table 1 defines the measure of social welfare maximized by the utilitarian social planner in each of these cases. SW_x refers to the measure of social welfare, where $x \in \{1A, 1B, 2A, 2B\}$. Individual welfare of consumer i is denoted by $W^i(.)$, which may depend on the true probability, $W^i(q^{true})$, or on her own belief, $W^i(q_i)$. Similarly, $\hat{W}^i(.)$ refers to the social planner's estimate for individual welfare of consumer i, which may depend on the estimate of the planner for the true probability, $\hat{W}^i(\hat{q}^{true})$, or on the estimate for the corresponding individual belief, $\hat{W}^i(\hat{q}_i)$, depending on the case.

Table 1: Social planner's objective function

	A. Social planner with complete information	B. Social planner with incomplete information
Case 1: Individual welfare depends on q^{true}	$SW_{1A} = \int_0^1 W^i(q^{true})di$	$SW_{1B} = \int_0^1 \hat{W}^i(\hat{q}^{true})di$
Case 2: Individual welfare depends on q_i	$SW_{2A} = \int_0^1 W^i(q_i)di$	$SW_{2B} = \int_0^1 \hat{W}^i(\hat{q}_i)di$

In contrast, the problem of consumer i does not change across cases since, in this environment, individuals always behave according to their beliefs. Formally, consumer i solves

$$\max_{d_i} \mathrm{E}[U^i] = (1 - q_i)\Big[\lambda u(c_E^i) + (1 - \lambda)u(c_L^i)\Big] + q_i u(c_R^i), \tag{10}$$

with

$$c_E^i = d_i \epsilon \frac{c_1^B}{D_0} + (1 - d_i)\epsilon(1 - f), \qquad c_L^i = d_i \epsilon \frac{c_2^B}{D_0} + (1 - d_i)\epsilon(1 - f),$$
$$c_R^i = d_i \epsilon \frac{c_R^B}{D_0} + (1 - d_i)\epsilon(1 - f),$$

where ϵ denotes individual consumer's endowment (which is identical across all consumers) and $d_i \in \{0, 1\}$ determines whether i places her endowment in public money or deposits. Importantly, actual welfare of consumer i may differ from her objective function or not depending on whether individual welfare depends on q^{true} or on q_i, respectively.

We study the individual and social welfare implications of introducing CBDC under each of these cases. In order to do so, we numerically solve the problem of individual consumers and of the social planner for all possible values of the relevant probability. In order to make conclusive statements on the impact of CBDC as a store of value on social welfare under this general approach, we rely on the "belief-neutral" welfare criterion proposed by Brunnermeier et al. (2014). An allocation is said to be belief-neutral superior in case it dominates under every reasonable belief or probability. Two allocations are said to be incomparable when one or the other dominates depending on the belief or probability.

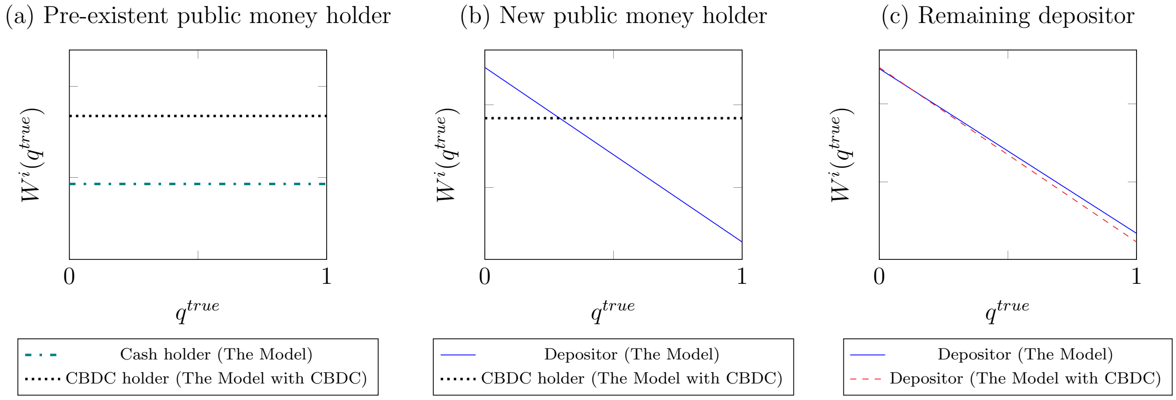
5.1 Case 1: Welfare depends on actual bank stability

We first consider the case in which individual welfare depends on the true probability of a bank run. Depending on their response to the introduction of CBDC, we differentiate between three types of agents: (i) consumers who remain as public money holders and fully replace cash with CBDC (i.e., pre-existent public money holders); (ii) consumers who switch from deposits to CBDC (i.e., new public money holders); and (iii) consumers who remain as depositors (i.e., remaining depositors).[17]

Figure 9 plots simulated individual welfare for each of the three types of consumers in The Model with and without CBDC, for the entire range of possible q^{true} values. Pre-existent public money holders benefit from the introduction of CBDC by fully switching from cash to the central bank digital currency. The increase in their welfare is proportional to the difference between cash and CBDC storage costs (Figure 9a). Such increase in the individual welfare of holding public money implies that the threshold that defines the set of consumers who prefer to hold public money decreases with CBDC (Proposition 5). Thus, some of the consumers who were holding deposits switch to CBDC based on their individual beliefs. These new public money holders are indeed better-off when the true probability of a bank run is sufficiently high (Figure 9b). As explained in Proposition 5, the change in the depositor base lowers (absolute) maturity transformation and productive investment. The feasibility set of the bank shrinks and, consequently, remaining depositors are worse-off with CBDC for a wide range of possible q^{true} values (Figure 9c).

[17] Theoretically, there could be a fourth type of agent; public money holders who replace cash with deposits. However, in our framework this agent type does not exist. No cash holder will ever switch to deposits when offered a more attractive public store of value.

Figure 9: Simulated individual welfare by consumer type in Case 1

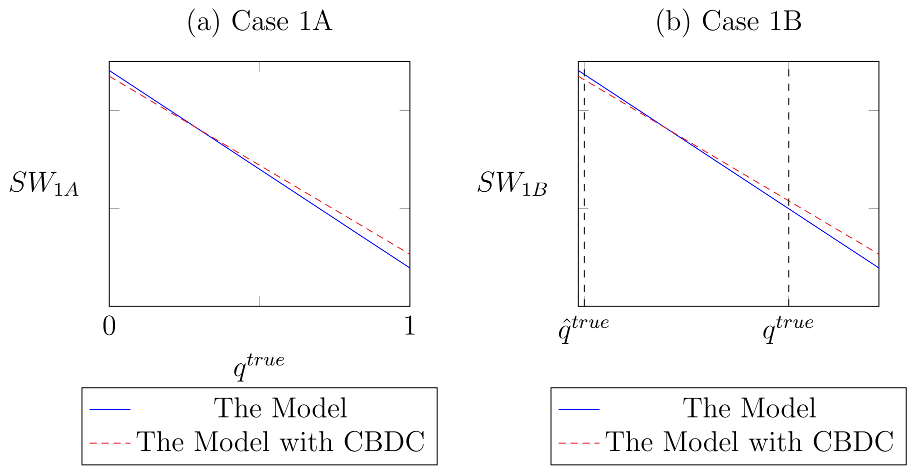


Notes: The Figure plots the simulated individual welfare under Case 1 in The Model with and without CBDC for each type of consumer and the entire range of q^{true} values when $R = 2.0$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$, and $q_i \sim Beta(1, 5)$. In this numerical example $D_0 = 0.96$ and $D_0^{DC} = 0.81$.

Under Case 1A, the social planner knows the value of q^{true}.[18] Given our assumptions and based on our simulations, the two allocations are incomparable as CBDC is social welfare increasing or decreasing depending on the actual probability of a bank run (see Figure 10a). In contrast, under Case 1B the social planner does not know the value of q^{true} and, therefore, has to estimate it. Thus, if the two allocations are incomparable and the estimation error is sufficiently large, the choice of the social planner under Case 1B may differ from that under Case 1A. Figure 10b illustrates a case in which, due to a significant underestimation of q^{true}, the planner chooses not to introduce CBDC as a store of value whereas the choice based on the maximization of actual social welfare would be to introduce it.

[18]This case could be interpreted as one in which a paternalist social planner knows what is best for society and does it regardless of consumers' individual behaviour.

Figure 10: Simulated social welfare and the planner's choice



Notes: The Figure plots the simulated social welfare under Case 1 in The Model with and without CBDC for the entire range of q^{true} values when $R = 2$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$ and $q_i \sim Beta(1, 5)$. In this numerical example $D_0 = 0.96$ and $D_0^{DC} = 0.81$.

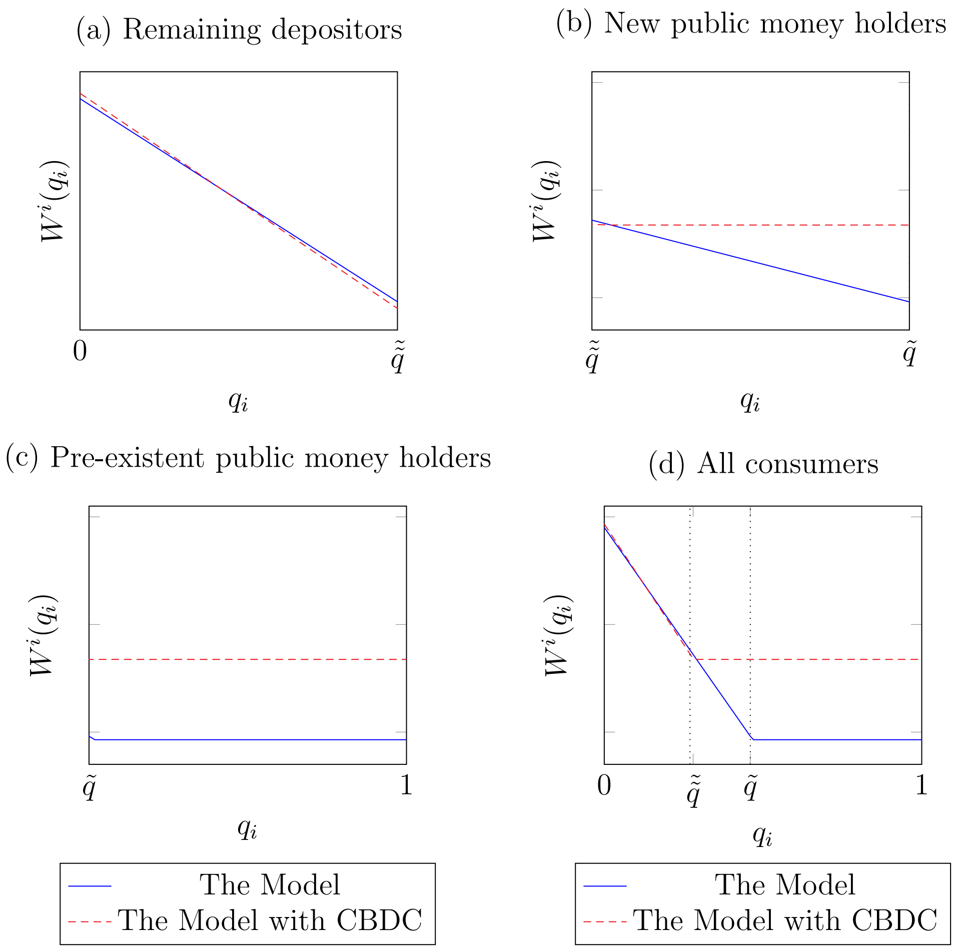
5.2 Case 2: Welfare depends on individual beliefs

Then, we consider the case in which individual welfare of consumer i depends on her own perceptions about bank stability, q_i. This implies that individual behaviour is unambiguously consistent with the maximization of individual welfare. Figure 11 plots simulated individual welfare for each of the three types of consumers in The Model with and without CBDC, by considering the entire range of possible q_i values. Individual welfare consequences vary across remaining depositors (i.e., those consumers for which $q_i < \tilde{\tilde{q}}$) depending on their beliefs (Figure 11a). The adoption of a CBDC leads to a decline in the average belief of depositors, on which the deposit contract offered by the bank depends. Therefore, CBDC increases individual welfare for all remaining depositors who were more optimistic about bank stability than the previous average depositor. All other remaining depositors are worse-off (see also Corollary 3). These consumers optimally choose to remain being depositors despite the welfare loss because, given their individual beliefs, the welfare loss of switching to CBDC would be larger.

Something similar happens for the case of those depositors who switch to CBDC. Some of the new public money holders are worse-off. However, they still prefer to switch to CBDC since - given their beliefs - the decline in the average belief of depositors (and the corresponding change in the deposit contract) makes them prefer to hold CBDC rather than deposits. All other new public money holders benefit from the introduction of CBDC. In this case, CBDC-induced individual welfare gains are increasing in the pessimism of the consumer (Figure 11b). In contrast, all pre-existent public money holders benefit by switching to CBDC. Their individual welfare gains are proportional to f and do not vary across consumers as they are independent from their individual beliefs

(Figure 11c).

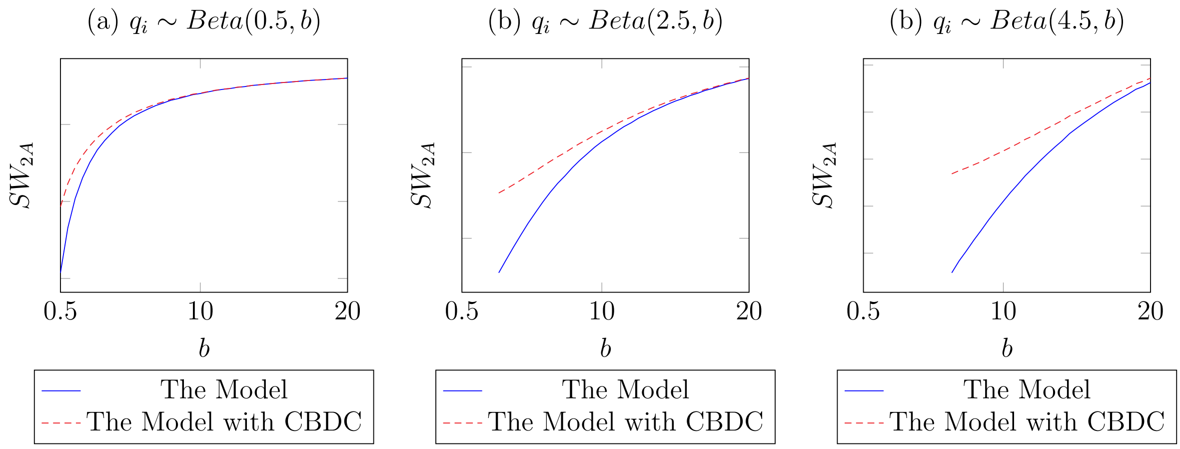
Figure 11: Simulated individual welfare by consumer type in Case 2



Notes: The Figure plots simulated individual welfare under Case 2 in The Model with and without CBDC for each type of consumer and the entire range of q_i values when $R = 2.0$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$, and $q_i \sim Beta(1, 5)$. In this numerical example $D_0 = 0.96$ and $D_0^{DC} = 0.81$.

Given the definition of social welfare and based on our simulations, the introduction of CBDC leads to an increase in social welfare as the majority of consumers are better-off with the availability of this new public storage technology (see Figure 11d). Interestingly, we were unable to find a distribution of individual beliefs for which CBDC is social welfare decreasing. Figure 12 illustrates this by plotting social welfare in The Model with and without CBDC for a wide range of Beta distributions. Thus, regardless of whether the social planner knows the distribution of individual beliefs (Case 2A) or not (Case 2B), in this general case the Model with CBDC constitutes a belief-neutral superior allocation.

Figure 12: Simulated social welfare for different distributions in Case 2



Notes: The Figure plots simulated social welfare under Case 2 in The Model with and without CBDC when $R = 2.0$, $\lambda = 0.3$, $\gamma = 1.5$, $f = 0.2$, $f^{DC} = 0$ and for a comprehensive range of individual belief distributions that accounts for the spectrum of cases for which the assumption of the run-prone deposit contract holds. In panels (b) and (c), the optimal deposit contract is the run-proof contract when the parameter b is below a cut-off point, in which case we do not plot welfare.

5.3 Uncertainty and welfare

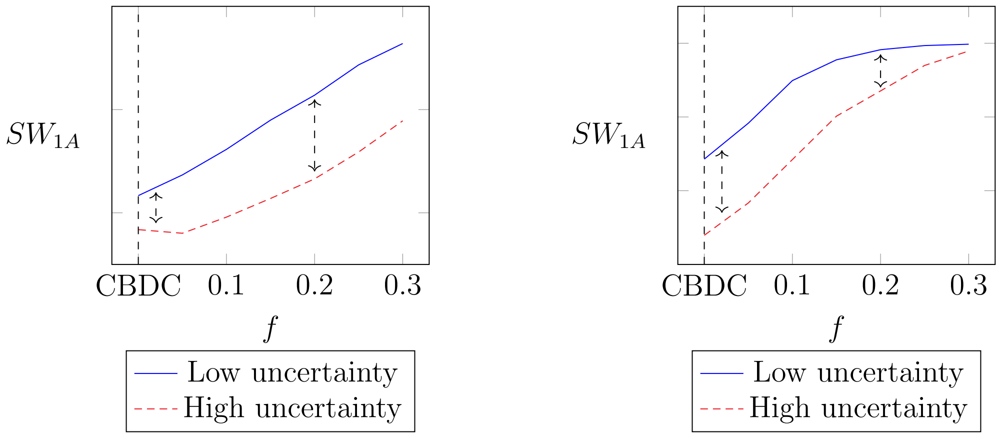
Next, we turn to the question of how does the availability of CBDC affect the impact of uncertainty shocks on social welfare. In order to do so, recall from Section 4.3 that the sign and scale of the excess flight-to-safety induced by CBDC in response to uncertainty shocks crucially depend on the distribution of beliefs as well as on the degree of technological superiority of CBDC, captured by parameter f.

Consider Case 1A. Given a relevant range of f values, Figure 13 displays simulated social welfare before and after the uncertainty shock for the distribution of beliefs used throughout this section (panel a) and for an alternative distribution associated with a larger share of depositors (panel b). Note that social welfare in The Model with CBDC corresponds to the intersection of the relevant social welfare curve with the vertical dashed line, at which $f = 0$. First, in these cases the uncertainty shock negatively affects social welfare.[19] Second, depending on whether CBDC mitigates or amplifies flight-to-safety, its adoption reduces (panel a) or increases (panel b) these welfare losses, respectively. For further details on why in this particular case CBDC mitigates flight-to-safety under the distribution considered in panel a and amplifies it under the distribution used in panel b, see Section 4.3 and Appendix D.

[19]This result may hold or not depending on the distribution of beliefs and the ranges of values considered for parameters f and q^{true}.

Figure 13: Simulated social welfare and uncertainty in Case 1A

(a) Initial distribution: $q_i \sim Beta(1, 5)$ (b) Initial distribution: $q_i \sim Beta(4, 10)$



Notes: The Figure plots simulated social welfare under Case 1 in The Model with and without CBDC for a range of f values and before and after an uncertainty shock when $\lambda = 0.3$, $\gamma = 1.5$, $f^{DC} = 0$, and $q^{true} = 0.05$. In Panel (a) $R = 2$, $q_i \sim Beta(1, 5)$ (low uncertainty) and $q_i \sim Beta(0.5, 2.5)$ (high uncertainty) and in panel (b) $R = 3$, $q_i \sim Beta(4, 10)$ (low uncertainty) and $q_i \sim Beta(2, 5)$ (high uncertainty).

To summarize, the adoption of CBDC as a store of value has heterogeneous welfare consequences across the population. Cash holders always benefit by fully switching to CBDC. Each of all other consumers may benefit or not depending on the true probability of a bank run, her (and all others') belief about such probability, and the degree of technological superiority of CBDC. Under the assumption that the social planner is utilitarian, CBDC as a store of value increases social welfare structurally in a wide range of cases and, depending on the distribution of individual beliefs, it may mitigate the welfare effects induced by flight-to-safety episodes.

6 Conclusion

This paper develops a banking model a la Diamond and Dybvig (1983) with public money as a store of value and heterogeneous beliefs about bank stability. The assumption of heterogeneous beliefs allows to rationalize how different consumers choose between risky bank deposits and cash holdings, which are safe but subject to storage costs. Our model accounts for key empirical regularities on cash hoarding by building on the literature about portfolio choice and belief disagreement. We use this model to study the implications of CBDC as a storage technology for banks and welfare.

Our positive analysis concludes that, under certain assumptions, CBDC lowers the cost of storing public money although it also induces partial bank disintermediation. The latter effect is nevertheless mitigated by an increase in relative maturity transformation which follows from the fact that remaining depositors are, on average, more optimistic.

CBDC may amplify or mitigate fligh-to-safety episodes depending on its design as well as on the distribution of beliefs.

Our normative analysis illustrates how CBDC has heterogeneous welfare consequences across the population. While cash holders always benefit by switching to CBDC, each of all other consumers may be better-off or not depending on the measure of bank stability on which welfare depends, her (and all others') subjective perceptions about bank stability, and the degree of technological superiority of CBDC. Depending on these factors and on the welfare criterion, CBDC as a store of value may increase social welfare or not. Similarly, depending on these factors, CBDC may also mitigate or amplify the welfare effects induced by uncertainty shocks.

Although beyond the scope of this paper, throughout this piece we refer to and discuss various potential extensions of our work which, in our view, constitute promising avenues for future research. Among others, the modification of the proposed model to allow for multiple deposit contracts (or multiple banks), heterogeneous belief-formation based on learning, narrow banking, endogenous probability of a bank run, or optimal deposit insurance.

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A Evidence: the store of value component of cash

Section 1 presents motivating evidence on the use of euro banknotes as a store of value. Panel (b) of Figure 1 displays the estimated component of total cash holdings used for transaction purposes and that used as store of value. Similar to Assenmacher et al. (2019); Zamora-Pérez (2021), the decomposition estimates are produced by comparing the seasonality of total banknote circulation with the seasonality of a purely transactional benchmark variable.

Formally, total cash demand can be modeled as the product of a trend-cycle component, T_t, and a seasonal component, S_t. This decomposition also holds for the domestic transactions (T_t^{tr}) and non-transaction or store of value related demand (T_t^{ntr}), respectively:

$$T_t S_t = T_t^{tr} S_t^{tr} + T_t^{ntr} S_t^{ntr}$$

Let β_t be the share of the overall trend-cycle component demanded for transaction purposes, and $1 - \beta_t$ be the share demanded as a store of value. This leads to

$$T_t S_t = \beta_t T_t S_t^{tr} + (1 - \beta_t) T_t S_t^{ntr}$$

This Equation can be simplified by cancelling T_t from both sides of the Equation and by assuming that the demand as a store of value does not exhibit seasonal variation, i.e., $S_t^{ntr} = 1 \forall t$:

$$S_t = \beta_t S_t^t r + (1 - \beta_t)$$

When solving for $(1 - \beta_t)$, this Equation yields the estimated share of cash demand as a store of value (by residents and non-residents):

$$(1 - \beta_t) = 1 - \frac{S_t - 1}{S_t^{tr} - 1}$$

The seasonal factor S_t can be extracted from total cash demand. The seasonal factors for the purely transactions-related series, S_{tr}, cannot be observed directly and have to be approximated through the use of a benchmark series. We proxy a purely transactions-related series using data on small denomination series and bank vault cash. To obtain more robust estimates we use a range of seasonal factors obtained from monthly data.

B Proofs

B.1 Proposition 1

For any value for q, the deposit contract with the lowest possible expected return offered by the bank is a run-proof contract that includes no long-term lending ($x = 0$). In that case, Problem (A) reduces to

$$\max_y \lambda u(\frac{y}{\lambda}) + (1 - \lambda) u(\frac{y^l}{1 - \lambda}),$$

subject to

$$y + y^l = D_0, \tag{B.11}$$
$$0 \leq c_1^B \leq c_2^B. \tag{B.12}$$

The first order condition that characterizes the solution is given by

$$u'(\frac{y^l}{1-\lambda}) - u'(\frac{D_0 - y^l}{\lambda}) = 0. \tag{B.13}$$

This defines $y^l = (1-\lambda)D_0$ and $y = \lambda D_0$.

If a single atomistic consumer with endowment ϵ invests in deposits, her expected utility is

$$E[U^{deposit}] = u\Big[\epsilon\Big].$$

If she instead holds cash, her expected utility is

$$E[U^{cash}] = u\Big[\epsilon(1-f)\Big].$$

Since $f > 0$, she will not hold any cash. Any chosen deposit contract yields at least as high expected returns and, thus, there is never any demand for cash.

B.2 Corollary 1

The proof consists of three parts: i) the run-proof solution to Problem (A); ii) the run-prone solution to Problem (A); iii) the conditions under which each contract is offered. We will use that there is no cash demand (Proposition 1), but we include deposit funding D_0 in the bank's problem as it is useful for the proof of Corollary 2.

First, an optimal run-proof contract solves Problem (A) where the indicator function is zero and subject to an additional constraint that allows only for run-proof contracts:

$$y^l = c_1^B - y. \tag{B.14}$$

Let η_E and η_L be the Lagrange multipliers on constraints (2) and (3) of Problem (A), respectively, and η_R the multiplier on the additional constraint (B.14). Let γ and β be the multipliers on the non-negativity constraints for x and y^l, respectively. When first ignoring the incentive compatibility constraint, the first order conditions that characterize the solution are given by

$$c_1^B : \lambda u'(c_1^B) - \eta_E \lambda - \eta_R = 0, \tag{B.15}$$
$$c_2^B : (1-\lambda)u'(c_2^B) - \eta_L(1-\lambda) = 0, \tag{B.16}$$
$$x : -\eta_E + \eta_L R - \eta_R + \gamma = 0, \tag{B.17}$$
$$y^l : -\eta_E + \eta_L + \beta = 0. \tag{B.18}$$

Rewriting (B.15) gives

$$\eta_E = u'(c_1^B) - \frac{1}{\lambda}\eta_R,$$

and rewriting (B.16) gives

$$\eta_L = u'(c_2^B).$$

Since $y^l > 0$ must hold for any run-proof contract, $\beta = 0$ and thus Expression (B.18) implies that $\eta_L = \eta_E$. This allows to solve for η_R as

$$\eta_R = \lambda\Big[u'(c_1^B) - u'(c_2^B)\Big].$$

Substituting for η_E, η_L, and η_R into Expression (B.17) gives the following optimality condition

$$u'(c_1^B) = u'(c_2^B)\frac{R-1+\lambda}{\lambda}. \tag{B.19}$$

Since $R > 1$ and u is concave, the optimal run-proof contract is indeed incentive compatible.

The optimality condition (B.19) is restated as

$$u'\Big[\frac{y}{\lambda}\Big] = \frac{R-1+\lambda}{\lambda}u'\Big[\frac{R(D_0-y)}{1-\lambda} - (R-1)\frac{y}{\lambda}\Big], \tag{B.20}$$

and when $D_0 = 1$ this results in a solution $y = y^{proof}$. We denote the resulting expected utility of a single consumer with endowment ϵ who deposits with the bank as

$$E[U^{proof}] = \lambda u\Big[\frac{\epsilon}{D_0}\frac{y^{proof}}{\lambda}\Big] + (1-\lambda)u\Big[\frac{\epsilon}{D_0}\Big[\frac{R(D_0-y^{proof})}{1-\lambda} - (R-1)\frac{y^{proof}}{\lambda}\Big]\Big].$$

Second, an optimal run-prone contract solves Problem (A) where the indicator function is equal to one. Let η_E and η_L be the Lagrange multipliers on constraints (2) and (3) of Problem (A), and let γ and β be the multipliers on the non-negativity constraints for x and y^l, respectively. When first ignoring the incentive compatibility constraint, the first order conditions that characterize the solution are given by

$$c_1^B : (1-q)\lambda u'(c_1^B) + q\lambda u'(c_R^B) - \eta_E\lambda = 0, \tag{B.21}$$
$$c_2^B : (1-q)(1-\lambda)u'(c_2^B) + q(1-\lambda)u'(c_R^B) - \eta_L(1-\lambda) = 0, \tag{B.22}$$
$$x : -qRu'(c_R^B) - \eta_E + \eta_L R + \gamma = 0, \tag{B.23}$$
$$y^l : -\eta_E + \eta_L + \beta = 0. \tag{B.24}$$

We will first show that any optimal run-prone contract has no excess liquidity. To do so, suppose the opposite, so that $y^l > 0$. Then $\beta = 0$ must hold. From (B.23) and (B.24), it follows that $\eta_E = \eta_L = \frac{qR}{R-1}u'(c_R^B)$, while from (B.22) we find that $\eta_L = (1-q)u'(c_2^B) + qu'(c_R^B)$. Combining these two expressions for η_L gives:

$$(R-1)(1-q)u'(c_2^B) = qu'(c_R^B).$$

From expressions (B.21) and (B.22), we also find that $c_1^B = c_2^B$. Thus, this implies the following relationship between c_1^B, c_2^B and c_R^B

$$c_1^B = c_2^B = Ac_R^B,$$

where constant A equals

$$A = [\frac{q}{(R-1)(1-q)}]^{-\frac{1}{\gamma}} > 0.$$

We can now rewrite the problem as

$$max_{c_1^B}(1-q)u(c_1^B) + qu(\frac{1}{A}c_1^B).$$

At the optimum, the following first order condition must apply

$$(1-q)u'(c_1^B) + q\frac{1}{A}u'(\frac{1}{A}c_1^B) = 0,$$

which is never satisfied since $A > 0$. Thus, we must have that $y^l = 0$ at the solution. Now, using that $y^l = 0$, the first order conditions reduce to

$$c_1^B : (1-q)\lambda u'(c_1^B) + q\lambda u'(c_R^B) - \eta_E\lambda = 0, \tag{B.25}$$
$$c_2^B : (1-q)(1-\lambda)u'(c_2^B) - \eta_L(1-\lambda) = 0, \tag{B.26}$$
$$x : -qRu'(c_R^B) - \eta_E + \eta_L R = 0. \tag{B.27}$$

Rewriting Expression (B.25) gives

$$\eta_E = (1-q)u'(c_1^B) + qu'(c_R^B),$$

and rewriting Expression (B.26) gives

$$\eta_L = (1-q)u'(c_2^B).$$

Substituting for η_E and η_L into Expression (B.27) gives

$$-qu'(c_R^B) - (1-q)u'(c_1^B) + (1-q)Ru'(c_2^B) = 0,$$

which can be rewritten into the optimality condition

$$(1-q)\Big[Ru'(c_2^B) - u'(c_1^B)\Big] = qu'(c_R^B). \tag{B.28}$$

When $q = 0$, and since $R > 1$, the contract that satisfies the optimality condition (B.28) is incentive compatible as $u'(c_2^B) < u'(c_1^B)$ and hence $c_2^B > c_1^B$. When $q > 0$, the optimality condition (B.28) is restated as

$$u'(\frac{y}{\lambda}) = Ru'(\frac{R(D_0-y)}{1-\lambda}) - \frac{q}{1-q}u'(y).$$

Using the assumed utility function, y is solved for as

$$y = \frac{\frac{RD_0}{1-\lambda}}{R^{\frac{1}{\gamma}}\left[\frac{1}{\lambda^{-\gamma}} + \frac{q}{1-q}\right]^{-\frac{1}{\gamma}} + \frac{R}{1-\lambda}}. \tag{B.29}$$

It follows that $\frac{\partial y}{\partial q} > 0$, and so $\frac{\partial c_1^B}{\partial q} > 0$ and $\frac{\partial c_2^B}{\partial q} < 0$. Thus, a critical bound q' exists such that when $q > q'$ it holds that the solution to the optimality condition (B.28) includes $c_2^B < c_1^B$ and when $q < q'$ it holds that $c_2^B > c_1^B$.

Since the solution to the optimality condition (B.28) when $q > q'$ contradicts with the incentive compatibility constraint, it cannot be an equilibrium contract. Instead, if $q > q'$ and the bank wishes to offer a run-prone contract, the best incentive compatible run-prone contract it could offer is the solution to optimality condition (B.28) when $y^* = y|_{c_1^B = c_2^B}$: the run-prone incentive compatible contract with the highest early repayment. Utility under this run-prone contract with $y = y^*$ of a single consumer who deposits her endowment ϵ with the bank is given as

$$E[U^{prone}|y = y^*] = (1-q)\Big[\lambda u(\frac{\epsilon}{D_0}\frac{y^*}{\lambda}) + (1-\lambda)u(\frac{\epsilon}{D_0}\frac{R(D_0 - y^*)}{1-\lambda}\Big] + qu(\frac{\epsilon}{D_0}y^*). \quad \text{(B.30)}$$

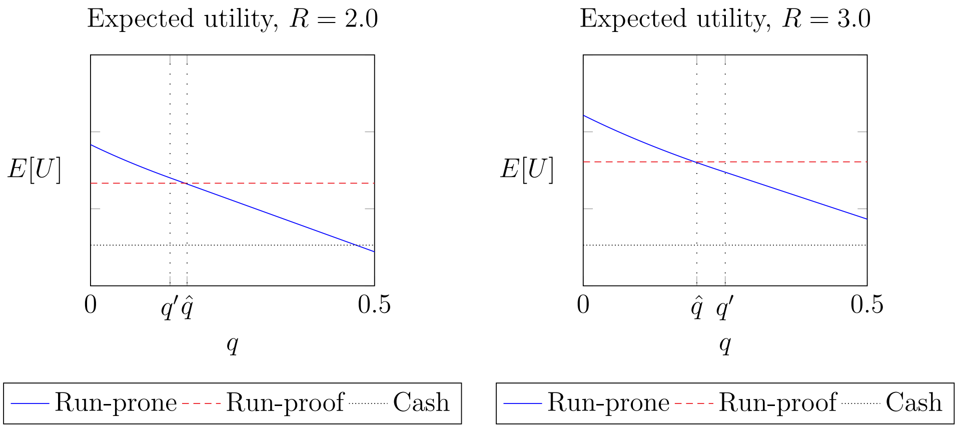
Finally, we solve for the unique equilibrium contract. From expressions (B.29) and (B.30) it follows that an optimal run-prone contract is such that $\frac{\partial U^{prone}}{\partial q} < 0$, both when $q < q'$ and when $q > q'$. Thus, we can derive a second critical bound $\hat{q}$ such that when $q = \hat{q}$ the utility obtained from the run-prone deposit contract is equal to the utility obtained from the run-proof contract. The cut-off value $\hat{q}$ is equal to

$$\hat{q} = \frac{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda})}{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda}) - u(\frac{1}{D_0}y)} - \dots$$
$$\frac{\lambda u\Big[\frac{1}{D_0}\frac{y^{proof}}{\lambda}\Big] + (1-\lambda)u\Big[\frac{1}{D_0}\Big[\frac{R(D_0-y^{proof})}{1-\lambda} - (R-1)\frac{y^{proof}}{\lambda}\Big]\Big]}{\lambda u(\frac{1}{D_0}\frac{y}{\lambda}) + (1-\lambda)u(\frac{1}{D_0}\frac{R(D_0-y)}{1-\lambda}) - u(\frac{1}{D_0}y)}.$$

where when $\hat{q} < q'$, y is the solution to optimality condition (B.28) using $q = \hat{q}$, and when $\hat{q} > q'$, y is the solution to (B.28) using $q = q'$ and $c_1^B = c_2^B$.

Figure (B.14) illustrates the relative cut-off values by plotting the simulated expected utility of each type of contract as a function of q, using that $D_0 = 1$ and under different values for R.

Figure B.14: Optimal deposit contract



The figure plots the expected utility when $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

B.3 Proposition 2

The run-prone contract offered by the bank satisfies Expression (9). A consumer's expected utility when depositing its endowment ϵ with the bank depends on its belief q_i and equals

$$E[U^{deposits}] = (1-q_i)\Big[\lambda u(\frac{\epsilon}{D_0}c_1^B) + (1-\lambda)u(\frac{\epsilon}{D_0}c_2^B)\Big] + q_i u(\frac{\epsilon}{D_0}c_R^B).$$

If a consumer instead holds cash, her expected utility is

$$E[U^{cash}] = u\Big[\epsilon(1-f)\Big].$$

From here it follows that positive cash demand requires $E[U^{deposits}] < E[U^{cash}]$, so when

$$q_i > \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})} = \tilde{q}.$$

B.4 Proposition 3

Cash demand equals

$$M_0 = \int_{\tilde{q}}^{1} f(q,\sigma)dq.$$

First, when $\tilde{q} > E[q_i]$ and σ increases, all else equal, cash demand increases as the tail of the distribution becomes fatter. Next, cash demand also depends on the bound $\tilde{q}$, given as

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})}.$$

At lower deposit funding D_0, the average belief of bank depositors $\bar{q}$ decreases as only relatively optimistic depositors remain, or $\frac{\partial \bar{q}}{\partial \sigma} < 0$. This implies that the optimal $\frac{y}{D_0}$, as determined by Expression (B.29) with $q = \bar{q}$, decreases (so $\frac{\partial \frac{y}{D_0}}{\partial \sigma} < 0$), and thus the bound $\tilde{q}$ is affected:

$$\frac{\partial \tilde{q}}{\partial \sigma} = \frac{\partial \frac{y}{D_0}}{\partial \sigma} u'(\frac{c_R^B}{D_0}) \frac{\Big[\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)\Big] - \frac{\bar{q}}{1-\bar{q}}\Big[u(1-f) - u(\frac{c_R^B}{D_0})\Big]}{\Big[\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})\Big]^2}.$$

The sign of this Expression depends on $\bar{q}$. When $\bar{q} < \tilde{q}$ it is negative, and when $\bar{q} > \tilde{q}$ it is positive. Clearly, since only consumers with $q_i < \tilde{q}$ hold bank deposits, it must be that $\bar{q} < \tilde{q}$ and $\frac{\partial \tilde{q}}{\partial \sigma} < 0$: an increase in σ decreases the bound $\tilde{q}$, further increasing cash demand.

B.5 Proposition 5

Consider $\tilde{q}$:

$$\tilde{q} = \frac{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(1-f)}{\lambda u(\frac{c_1^B}{D_0}) + (1-\lambda)u(\frac{c_2^B}{D_0}) - u(\frac{c_R^B}{D_0})}.$$

Holding bank pay-outs and deposits constant, the impact of cash storage cost equals

$$\frac{\partial \tilde{q}}{\partial f} = \frac{u'(1-f)}{\lambda u(c_1^B) + (1-\lambda)u(c_2^B) - u(c_R^B)} > 0.$$

Thus, a decrease in cash storage cost, all else equal, results in a decrease of $\tilde{q}$.

Next, a lower $\tilde{q}$ implies an increase in M_0 and, thus, a decrease in D_0. Lower deposit funding not only implies lower reserves and lower long-term lending, but also a lower share of reserves in the bank's portfolio as $\bar{q}$ decreases (similar to in Proposition 3).

B.6 Corollary 3

From Problem A and Corollary 1, it follows that the return on deposits offered by the bank in case of a run-prone contract is given as

$$\frac{c_1^B}{D_0} = \frac{y}{\lambda D_0}$$

and

$$\frac{c_2^B}{D_0} = \frac{R(D_0 - y)}{(1-\lambda)D_0}$$

Proposition 5 states that the fraction $\frac{y}{D_0}$ decreases after the introduction of CBDC. Thus, $\frac{c_1^B}{D_0}$ decreases and $\frac{c_2^B}{D_0}$ increases.

C Mixed portfolios

C.1 Baseline model

Let c_E denote the consumption of impatient depositors (who consume at $t = 1$), c_L the consumption of patient depositors (who consume at $t = 1$ or $t = 2$) in case of no bank run, c_R the consumption in case of a bank run. Consider a single atomistic consumer who considers holding a share $d_0 \in [0, 1]$ of her endowment ϵ as deposits and a share $(1 - d_0)$ as cash. Her portfolio allocation problem, given a run-prone deposit contract, is given by

$$\max_{d_0} (1-q)\Big[\lambda u(c_E) + (1-\lambda)u(c_L)\Big] + qu(c_R),$$

subject to

$$c_E = d_0\epsilon\frac{c_1^B}{D_0} + (1-d_0)\epsilon(1-f), \qquad c_L = d_0\epsilon\frac{c_2^B}{D_0} + (1-d_0)\epsilon(1-f),$$
$$c_R = d_0\epsilon\frac{c_R^B}{D_0} + (1-d_0)\epsilon(1-f).$$

The first order condition for a given deposit contract equals

$$\begin{aligned}\lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1-f))\Big] + (1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1-f))\Big] + ...\\ q\Big[u'(c_R)\Big[\epsilon(\frac{c_R^B}{D_0} - (1-f))\Big] - \lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1-f))\Big] - ...\\ (1-\lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1-f))\Big]\Big] = 0.\end{aligned} \tag{C.31}$$

Expression C.31 can result in either corner solution or an interior choice for d_0, depending on q. Certainly, when q is sufficiently low, consumers only hold deposits.

However, the deposit contract is affected by deposit funding. The bank's problem in the version of the baseline model that allows for mixed portfolios is given by

$$\max_{c_1^B, c_2^B, c_R^B, x, y, y^l} (1 - \bar{q}\mathbb{1}_{\bar{\lambda}<1})\Big[\lambda u(c_E) + (1-\lambda)u(c_L)\Big] + \bar{q}\mathbb{1}_{\bar{\lambda}<1}u(c_R),$$

subject to

$$\begin{aligned} &x + y + y^l = D_0, && \lambda c_1^B = y,\\ &(1-\lambda)c_2^B = Rx + y^l, && c_R^B = y + y^l,\\ &0 \le c_1^B \le c_2^B, && c_1^B, c_2^B, x, y, y^l \ge 0,\end{aligned}$$

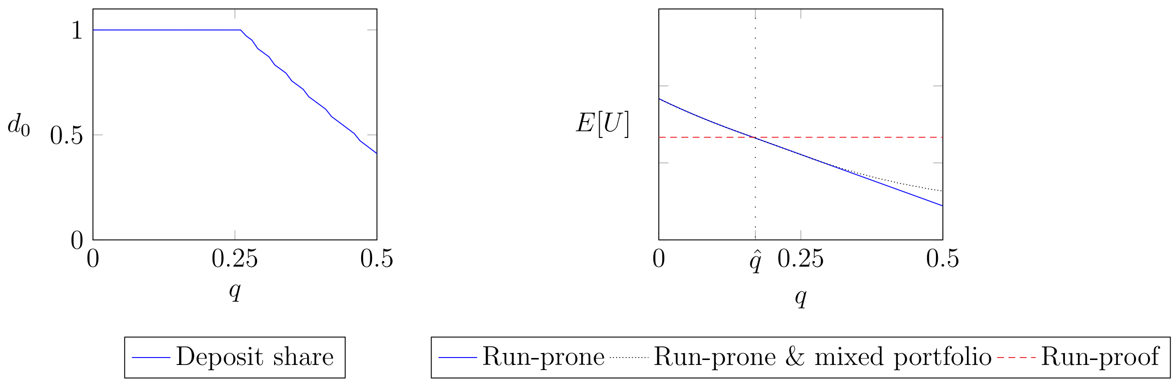
where

$$c_E = c_1^B + (1-f)(1-D_0), \quad (C.32) \qquad c_L = c_2^B + (1-f)(1-D_0), \quad (C.33)$$
$$c_R = c_R^B + (1-f)(1-D_0), \quad (C.34) \qquad \bar{\lambda} = \frac{y+y^l}{c_1^B}. \quad (C.35)$$

With an intermediate q, consumers may opt for some cash holdings (a mixed portfolio) in which case aggregate deposit funding D_0 would be lower and the bank optimally offers adjusted payoffs, similar to in Ennis and Keister (2003). In other words, in equilibrium the consumer's problem and the bank's problem are simultaneously determined. Panel (a) of Figure (C.15) uses a simulation to illustrate the run-prone contract consumers may choose for a mixed portfolio at an intermediate q. Panel (b) confirms that at some intermediate q a mixed portfolio results in a higher expected utility, given that the contract is run-prone. Panel (b) also shows that, under this calibration, the run-proof contract is always preferred over a linear combination of the run-prone contract and cash holdings and. That is, in this case, the same results apply regardless of whether mixed portfolios are allowed or not.

Figure C.15: Baseline model mixed portfolio

(a) Portfolio choice (b) Expected utility



Notes: In the simulations $R = 2.0$, $\lambda = 0.3$, $f = 0.2$, and $\gamma = 1.5$.

C.2 The Model

Consider a single atomistic consumer who considers holding a share d_0 of her endowment ϵ as deposits and a share $(1 - d_0)$ as cash. Her portfolio allocation problem given a run-prone contract depends on her belief, and is given by

$$\max_{d_0} (1 - q_i)\Big[\lambda u(c_E) + (1 - \lambda)u(c_L)\Big] + q_i u(c_R),$$

subject to

$$c_E = d_0\epsilon\frac{c_1^B}{D_0} + (1 - d_0)\epsilon(1 - f), \qquad c_L = d_0\epsilon\frac{c_2^B}{D_0} + (1 - d_0)\epsilon(1 - f),$$
$$c_R = d_0\epsilon\frac{c_R^B}{D_0} + (1 - d_0)\epsilon(1 - f).$$

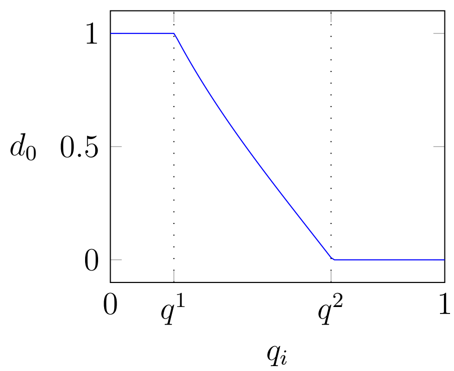
The first order condition equals

$$\begin{aligned}\lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1 - f))\Big] + (1 - \lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1 - f))\Big] + ...\\ q_i\Big[u'(c_R)\Big[\epsilon(\frac{c_R^B}{D_0} - (1 - f))\Big] - \lambda u'(c_E)\Big[\epsilon(\frac{c_1^B}{D_0} - (1 - f))\Big] - ...\\ (1 - \lambda)u'(c_L)\Big[\epsilon(\frac{c_2^B}{D_0} - (1 - f))\Big]\Big] = 0. \end{aligned} \tag{C.36}$$

The first term is positive and increasing in d_0, whereas the second term is negative and decreasing in d_0. Thus, a q^1 exists such that when $q_i < q^1$ it follows that $d_0 = 1$, a $q^2 > q^1$ such that when $q^1 < q_i < q^2$ it follows that $0 < d_0 < 1$ where d_0 solves optimality condition (C.36), and when $q_i > q^2$ it follows that $d_0 = 0$. Figure (C.16) uses a simulation to illustrate how consumers with an interior belief choose a mixed portfolio.

Figure C.16: Mixed portfolio

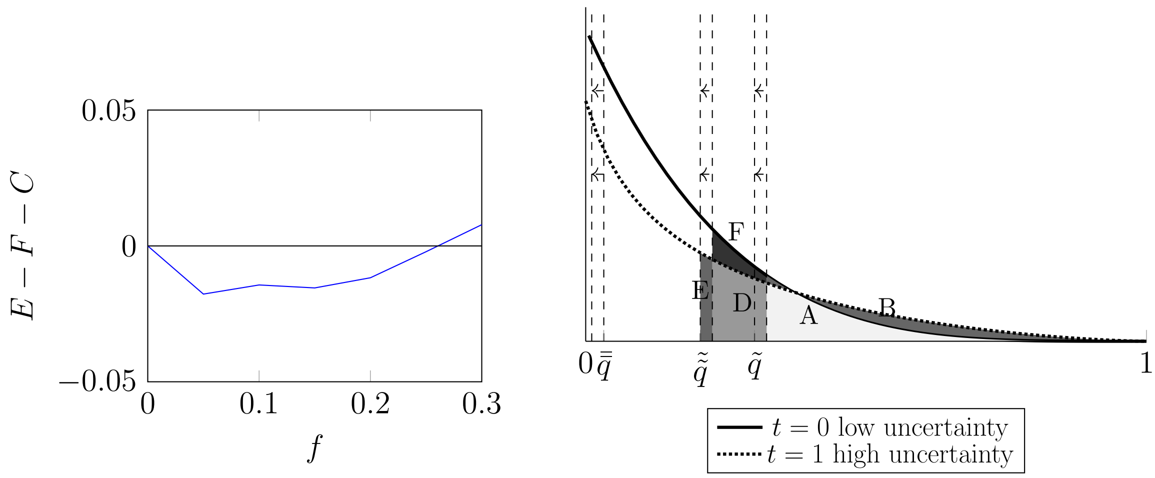
Share of endowment invested in deposits



Notes: The figure plots the portfolio choice when $R = 2.0$, $\lambda = 0.3$, $f = 0.2$, $\gamma = 1.5$, $D_0 = 0.7$, $y = 0.28$, and $\bar{q} = 0.05$.

D CBDC and flight-to-safety

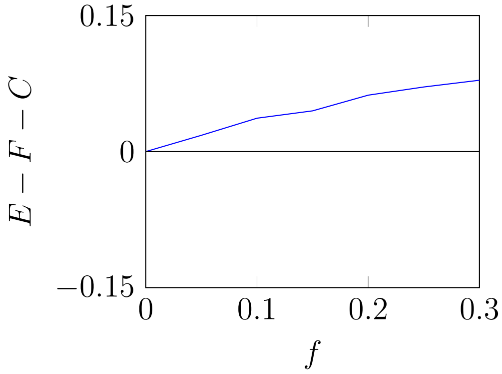
Figure D.17: CBDC mitigates flight-to-safety



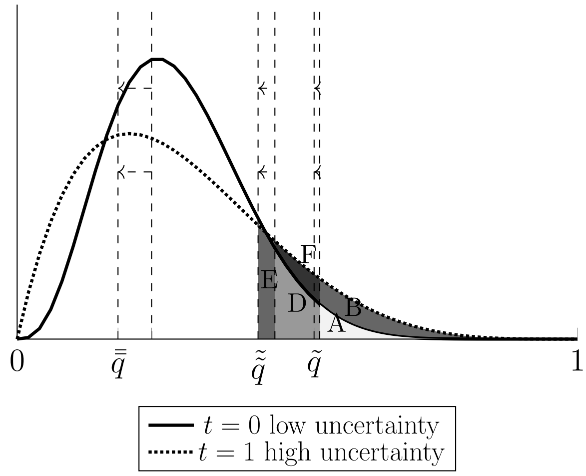
(a) CBDC impact on flight-to-safety (b) Public money demand when $f = 0.1$.

Notes: Panel (a) plots the difference in the simulated flight-to-safety in The Model with and without CBDC for a given mean-preserving shock to uncertainty and for a range of cash storage costs and for different distributions of individual beliefs when $\lambda = 0.3$, $\gamma = 1.5$, $f^{DC} = 0$, $R = 2.0$, $q_i \sim Beta(1, 5)$ (low uncertainty) and $q_i \sim Beta(0.5, 2.5)$ (dotted line, high uncertainty). Panel (b) summarizes the simulated effect for the particular case when $f = 0.1$, in which case $D_0 = 0.89$, $D_1 = 0.84$, $D_0^{DC} = 0.81$, $D_1^{DC} = 0.77$.

Figure D.18: CBDC amplifies flight-to-safety



(a) CBDC impact on flight-to-safety



(b) Public money demand when $f = 0.1$.

Notes: Panel (a) plots the difference in the simulated flight-to-safety in The Model with and without CBDC for a given mean-preserving shock to uncertainty and for a range of cash storage costs and for different distributions of individual beliefs when $\lambda = 0.3$, $\gamma = 1.5$, $f^{DC} = 0$, $R = 3.0$, $q_i \sim Beta(4, 10)$ (low uncertainty) and $q_i \sim Beta(2, 5)$ (dotted line, high uncertainty). Panel (b) summarizes the simulated effect for the particular case when $f = 0.1$, in which case $D_0 = 0.97$, $D_1 = 0.93$, $D_0^{DC} = 0.93$, $D_1^{DC} = 0.83$.

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